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Investigating the Domain of Geometric Inductive Reasoning Problems: A Structural Equation Modeling Analysis

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INVESTIGATING THE DOMAIN OF GEOMETRIC INDUCTIVE REASONING PROBLEMS: A STRUCTURAL EQUATION MODELING ANALYSIS

by

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As chair of the candidate's graduate committee, I have read the dissertation of Kairong Wang in its final form and have found that (1) its format, citations, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the graduate committee and is ready for submission to the university library.

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ABSTRACT

INVESTIGATING THE DOMAIN OF GEOMETRIC INDUCTIVE REASONING PROBLEMS: A STRUCTURAL EQUATION MODELING ANALYSIS

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Matrix inductive reasoning has been a popular research topic due to its claimed relationship with the general factor of intelligence. In this research, four subabilities were identified: working memory, rule induction, rule application, and figure detection. This quantitative study examined the relationship between these four subabilites and students' general ability to solve Matrix Reasoning problems. Using tests developed for this research to measure the identified subabilities, the data were collected from 334 Chinese students aged from 12 to 15. Structural equation modeling method was used to analyze the collected data and to evaluate the hypothesized models.

Results from the analysis showed that a valid model existed to represent the construct of matrix inductive reasoning. Except for figural detection ability, the other three subabilities had significant direct effects on matrix inductive reasoning ability. Readers should interpret from this result with caution due to the unsatisfactory reliability of the Figure Detection scores.

To improve the validity of the interpretation, a new model without the latent variable of figure detection was reexamined. In this analysis, significant relationships still existed from the three subablities to matrix inductive reasoning ability. The strongest relationship existed from working memory ability to matrix reasoning ability, with a standardized coefficient of .52. Effects from rule induction and rule application ability to matrix reasoning dropped to .36 and .34 respectively. These results suggested the important role of working memory on solving inductive reasoning problems. In addition, a significant and substantial indirect path was found that lead from working memory → rule induction → rule application → matrix reasoning. The indirect path indicated that a process existed when students solved Matrix Reasoning tasks.

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Chapter 1: Introduction

Need for Research on Matrix Inductive Reasoning

Human intelligence studies show that human abilities are strongly correlated. Of these abilities, a dominant factor which Spearman (1904) labels as *g* for general intelligence exist; the *g* factor influences the performance of all cognitive tasks. Lohman (2001) suggests that ". . . to understand essential aspects of what *g* might be and measure it clearly, we can start by understanding and measuring inductive reasoning abilities" (p. 220). After reviewing literature concerning cognitive tasks, Sternberg (1986) also concludes that "reasoning ability appears to be central to intelligence" (pp. 309-310). The fundamental position of reasoning ability has also been confirmed by a set of tests that were developed to examine inductive reasoning ability; these tests are known as Raven's Progressive Matrices Tests. In a summary scaling of several ability tests and learning tasks, Raven's test ranked directly in the center (Marshalek, Lohman & Snow, 1983). Inductive reasoning, therefore, could be the starting point for intelligence research. Studies on inductive reasoning help researchers gain a deeper understanding of human intelligence, which in turn may be used to find practical ways to improve human learning performance.

Matrix inductive reasoning is a form of analogical reasoning that involves inducing the rule or rules which govern the arrangement of geometric figures organized in rows and columns according to some predictable pattern. One cell of the matrix is deliberately left blank. The task of the examinee is to make an inference about which figure should be placed in the blank cell to best complete the observed pattern. .Matrix inductive reasoning tests are popular instruments for conducting research because of their nonverbal and culture-free characteristics studies on matrix inductive reasoning are primarily based upon Raven's Progressive Matrices. Many of these studies have focused on the structure and psychometric characteristics of the matrix tasks or on cross-cultural comparisons (Arendasy, 2005; Carpenter, Just & Shell, 1990; Mulholland, Pellegrino & Glaser, 1980; Primi, 2002; Sternberg, 1986). However, for educational purposes, in addition to research on item and test analysis, practical methods to improve performance in solving reasoning problems such as what abilities are required and how these abilities are related must be addressed. These questions involve investigations of the domain theory of inductive reasoning.

The term *domain theory,* as applied in educational measurement, was first used by Messick (1995). He claimed that "A major goal of domain theory is to understand the construct-relevant source of task difficulty, which will then serve as a guide to the rational development and scoring of performance tasks and other assessment formats" (p. 112). Bunderson (2002) extended the concept of domain theory by stressing the importance of developing theory simultaneously with assessment instruments and procedures. In Bunderson's (2003) description, he points out that "a domain theory gives an account of both sides of the person/item map–the substantive processes employed by the persons, and the construct-relevant sources of task difficulty" (p. 1). Bunderson addressed the need for studies on the person side of a person/item map to develop a domain theory.

Since the majority of previous research studies have mainly focused on the test/item characteristics, this research will examine the personal side that Bunderson addressed in domain theory. This study will further investigate the nature of the abilities required to solve Matrix Reasoning problems and the relationships among these abilities.

Background

In general, inductive reasoning is a process of drawing conclusions based on observations and a hypothesis. It may involve applying existing knowledge to predict a new instance in real life. Among the various tests which measure inductive reasoning ability, Raven's Progressive Matrices are consensually accepted as the quintessential test of inductive reasoning (Alderton & Larson, 1990).

The format of items in Raven's test is a geometric reasoning problem. Matrix tasks are visual analogy puzzles; each matrix task usually consists of several figures arranged in rows and columns with the last part missing. Corresponding figures or figural parts are organized according to a certain rule. The dimensions of each matrix can be 2 by 2, 2 by 3, 2 by 4, 3 by 3, or larger. In these entries, geometric shapes, lines, and background textures vary in form, number, orientation, and color. More than one rule may be used in the figures. Students must identify the existing relationship in the complete rows or columns, and then use that relationship to infer the missing entry in a new row. Using a variety of shapes, figure combinations, or rules, items with different difficulties can be created. Figure 1 is an example of Matrix Inductive Reasoning task.

The Raven's test was developed to measure two complementary components of general intelligence. Raven's two components include (a) the ability to think clearly and make sense of complex data, which is known as eductive ability, and (b) the ability to store and reproduce information, known as reproductive ability. Researchers have identified that Raven's tests are the most *g*-loaded of existing intelligence tests. Since

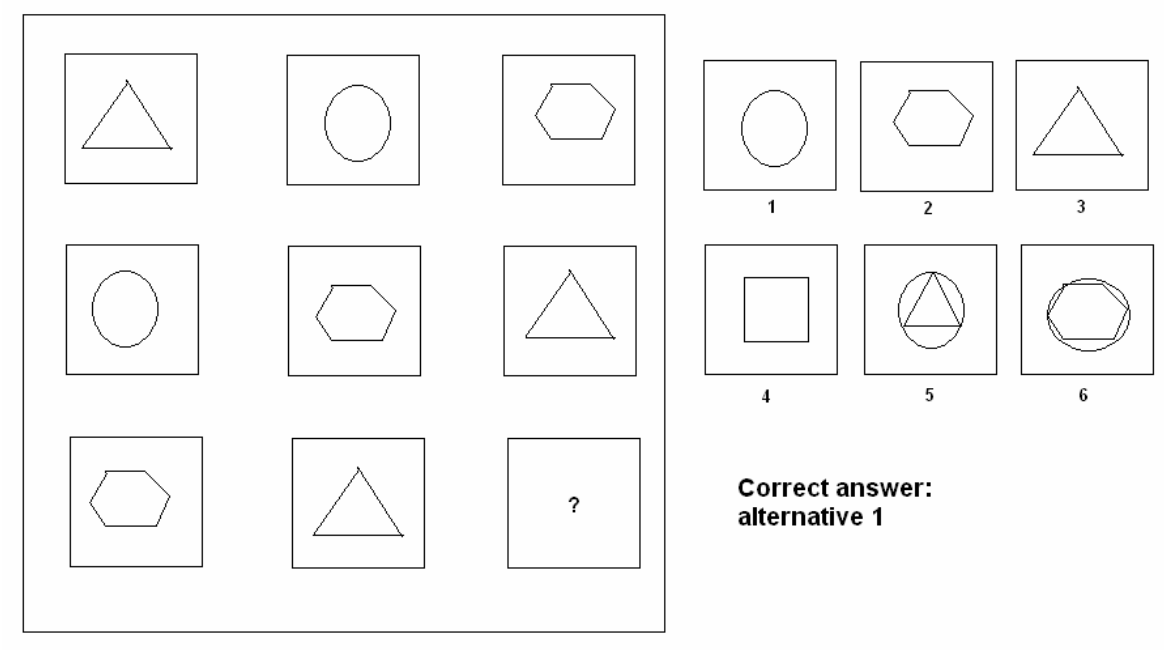


Figure 1. Example of a Matrix Inductive Reasoning task

Spearman has defined general intelligence as a unidimensional construct, and Raven's tests have been used to measure *g*, many have made the assumption that Raven's tests are unidimensional, meaning that they measure only one kind of ability. However, if there is only one kind of ability, what is this ability? How may one achieve it? Carpenter, Just, and Shell' (1990) research provided an alternative answer. In their research, Carpenter et al. found the following process is required to solve Raven's test problems: visual encoding, finding the rule, and goal management (managing problem-solving goals in the working memory). Based on an accumulation of data, the aforementioned Matrix Reasoning problems, and literature reviews, colleagues at the Edumetrics Institute identified four abilities that are needed to solve Matrix Inductive Reasoning problems. These include the ability to (a) decompose the figure into parts, (b) find the rules, (c) apply the rules, and (d) remember previous steps. The question of how these specific component abilities work in combination with each other to produce matrix inductive reasoning is the focal issue of this research.

Rationale for This Study

Although Raven's test has received more attention than other matrix reasoning tests (Arendasy, 2005; Carpenter et al., 1990; Embretson, 1995; Green et al., 2001; Hornke & Habon, 1986; Mulholland et al., 1980), scholars have still not obtained consistent results on important issues of its component constructs.

Most scholars have used factor analysis to explore the underlying structure of Raven's test. There is widespread disagreement over its constructs; some scholars conclude that it is a unidimensional test, and that the only ability it measures is the intelligence factor of *g*. Other scholars, on the other hand, claim that two or three factors have been found either by exploratory factor analysis or by confirmatory factor analysis techniques. These factors include (a) Pattern Addition/Subtraction and Detection of Pattern Progression, and (b) Verbal-Analytic and Visuospatial abilities (some rules need to be found by verbal analysis and others need to be induced by visuospatial analysis).

However, using factor analysis method to explore the structure of Raven's test should be cautioned because the collected dichotomous data can create a difficulty factor. As Gorsuch (1983) pointed out, categorical data with similar splits will necessary tend to correlate with each other, regardless of their content. The correlation reflect similarity of item difficulty. Hence the factor is called a difficulty factor. Because Raven's items are dichotomously scored and ranked across a wide range of difficulty levels, the items in similar difficulty level tend to cluster together. These factors are the dimensionality of the tasks difficulty but not the dimension of cognitive traits. Therefore, Rost and Gebert (1980) concluded that Raven's test items clustered according to their item difficulty but not due to additional relevant cognitive factors that influence performance on these items.

An investigation of the personal aspect of matrix reasoning domain theory would be helpful in answering the following questions: What abilities are required to solve a Matrix Reasoning problem? What are the interrelationships among these subabilities? Does a valid model exist that can explain one's performance in solving Matrix Reasoning problems?

A possible methodology to answer these questions is to first identify the abilities used to solve the Matrix Reasoning problem and then to conduct a CFA to test the results. From a synthesis of previous research on the cognitive analysis of Raven's test (Carpenter et al., 1990; Embretson, 1995; Hunt, 1974; and Jacobs & Vandeventer, 1972) and practical experience by a research team at the EduMetrics Institute, four ability factors which affect individual differences in Matrix Reasoning problems have been identified (a) figural decomposition ability, (b) rule induction ability, (c) deduction ability, and (d) working memory capacity. A series of hypotheses on the relationship of these will be proposed and empirically tested. The structural equation modeling (SEM) method will be used to test these hypothesized models.

Purpose and Research Questions

Based on this review, the main purpose of this project was to further investigate the process used to solve Matrix Reasoning problems. This research addressed the following questions:

1. Which of several alternative models is the best representation of the domain of Matrix Reasoning problem solving?
2. What modifications can be made to improve the model?
3. What are the significant direct and indirect effects of latent variables?

The predicted results of these questions indicate the existence of a valid model of matrix inductive reasoning ability. This model can help designers design better ways to assess progress and improvement in this sort of thinking, which can in turn help students to diagnosis problems they experienced when given a matrix problem.

The significance of this research lies in the investigation of the domain theory of matrix inductive reasoning from a cognitive process view, which will clarify which abilities are needed to solve these matrix tasks. It will further assist in developing instruments to improve performance in solving Matrix Inductive Reasoning problems.

Chapter 2: Review of the Literature

This study is an effort to understand the domain theory of matrix inductive reasoning from an empirical study. Literature review on previous studies will address how the domain of matrix inductive reasoning has been studied and what the connections are between past studies and the questions raised in this research.

Issues Pertaining to a Domain Theory

To understand the concept of domain theory, we must first define the concept *domain*. According to McShane (1991), a domain denotes "a collection of tasks that share a common representation system and a common set of procedures for operating on these representations to perform tasks" (p. 256). Thus, tasks which share common representation systems and common problem solving processes may be considered a domain. For example, *number series completion* is a domain of inductive reasoning, as are *verbal analogies* and *geometric analogies*. In this work, when we speak of the domain of matrix inductive reasoning we are referring to a broad collection of reasoning tasks, that spans a variety of stimulus formats and difficulty levels that all involve drawing inferences about the characteristics of a missing geometric figure in the context of a particular pattern that the examinee is expected to observe.

The concept of domain theory was first used by Messick (1995) to define construct validity:

> A major goal of domain theory is to understand the construct-relevant sources of task difficulty, which then serves as guide to the rational development and scoring of performance tasks and other assessment

> formats. At whatever stage of its development, the domain theory is a primary basis for specifying the boundaries and structure of the construct to be assessed. (p. 745)

Bunderson (2003) has broadened the concept of domain theory in the realm of human learning and instruction:

> Domain Theory (or learning theory of progressive attainments) is a descriptive theory of the contents, substantive processes, dimensional structure, and boundaries of a domain of human learning or growth that give an account of construct-relevant sources of task difficulty, and conjointly, an account of the substantive processes operative in persons at different levels of learning or growth along the scale(s) that span the domain. (p. 1)

This definition expands Messick's notion of domain theory as the boundaries and structure of a construct set by adding multiple dimensions and thinking processes. It also requires the assessment instrument to be associated with learning by stage (progressive attainments). At this point, a domain theory has connected tasks, processes, and learning locations along one or more measurement the same scales. Literature on aspects of the matrix inductive reasoning domain theory will be reviewed in the following sections.

General Introduction to Matrix Inductive Reasoning

Matrix inductive reasoning is a task type used to measure inductive reasoning ability. It is designed by following the central idea of inductive reasoning: reaching a general conclusion or overall rule based on limited observations. Raven's series progressive matrices are the most prominent examples of this type of test and are the most widely used non-verbal intelligence tests. Due to the high loading of its items on the general *g* factor, the Raven's test is considered to be one of the most *g*-loaded tests in existence.

Matrix Inductive Reasoning items are composed of figures. Subjects are asked to determine the patterns shown in these figures and infer the missing figure by applying the pattern to a new situation. Items are organized as 2 by 2, 3 by 3, 2 by 3, or 2 by 4 matrices. Generally the last entry of the matrix is generally empty, requiring the subject to deduce the answer. The components of figures in each entry include geometric shapes, shade, lines, and backgrounds. For the colored Raven's test, color is another component of the figures. These components vary in amount, form, color, position, and orientation in entries along the same row or column. Figure 1 is an example of a 3 by 3 inductive reasoning matrix.

In this example, there are 9 cells in the matrix with the last cell is empty. The subject is required to select an answer from the options provided. The subject must determine the relationship of components in the rows (or columns). For this example, we can see that there are different shapes—triangle, circle, and hexagon—distributed in the first two rows and the first two columns. The last entry is missing. From this observation, we can hypothesize that rule governed in each line or column is a distribution of three different shapes. Based on this hypothesized rule, the last entry should be one among the three shapes which is different from the other two shapes in the last row and the last column. Thus, the only option for the last entry is the circle. Therefore, option 1 is the correct answer.

As we go through the process of solving a matrix problem, we notice that one of the most important steps is finding the relationships or rules that govern the item. Researchers have done substantial work in exploring possible rules to develop these matrix questions (Arendasy, 2005; Carpenter et al., 1997; Hornke & Habon, 1986; Jacobs & Vandeventer, 1972; Primi, 2002; Ward & Fitapatrick, 1973). A list of selected rules which has been used in the past by researchers is listed in Table 1.

Item Difficulty Resources of Matrix Inductive Reasoning Problems

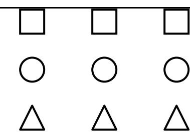
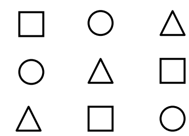
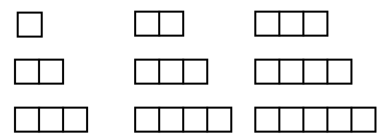
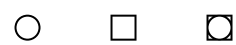
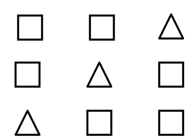
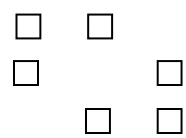
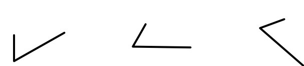
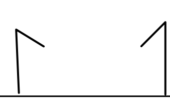
In order to design and develop different sources and levels of difficulty, the first task is to discover complexity factors underlying tasks. Studies on what characteristic of the items determines the item difficulty have been widely conducted. Matrix Inductive Reasoning item difficulty has been studied from the views of the problem solving process, the design experiment, and psychometric model analysis.

As Lohman (2002) points out that "Understanding what makes a task difficult is not the same as understanding how participants solve items on the task, but it is a useful place to start" (p. 225). The following researchers analyzed task difficulty by starting from an analysis of the inductive reasoning problem solving process.

According to an analysis by Carpenter et al. (1990) of the process of problem solving, the processes that distinguish individuals are primarily the ability of goal management and the ability of rule inducing. *Goal management* is the management of a large set of information in working memory. *Rule inducing* ability refers to discovering the rules that correspond to the figures and it is influenced by the rule type. Carpenter et al. also found that the error rate on a given problem was related to the types of rules and the number of rules involved. A simple conclusion based on the work of Carpenter et al. is that

Table 1.

Rules for the Solutions of Solving Matrix Inductive Reasoning Problems

Rule	Taxonomy	Example
Constant in a row	The same value occurs throughout a row, but changes down a column.	
Distribution of three values	Three values from a categorical attribute (such as figure type) are distributed through a row	
Quantitative pairwise progression	A quantitative increment or decrement occurs between adjacent entries in an attribute such as size, position, or number	
Figure addition or subtraction	A figure from one column is added to (juxtaposed or superimposed) or subtracted from another figure to produce the third	
Distribution of two plus one	Two same values and one different from a categorical attribute are distributed through a row (or column)	
Distribution of two values	Two values from a categorical attribute are distributed through a row (or column); the third value is null.	
Shading	Change may be complete or partial	
Size	Proportionate change, as in photographic enlargement	
Movement in a plane	Figure moves as if slid along surface	
Flip-over	Figure moves as if lifted up and replaced face down	

(table continues)

Table 1 (continued)

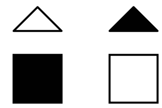
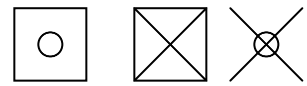
Rule	Taxonomy	Example
Reversal	Two elements exchange some feature, such as size, shading , or position	
Unique addition	Unique elements are treated differently from common elements, e.g., they are added while common elements cancel each other out	

figure decomposition ability, rule induction ability, and the hierarchy of goals management ability account for individual differences in performance in solving geometric problems in the Raven's test.

Mulholland, Pellegrino, and Glaser (1980) constructed 460 true-false analogies with varying numbers of elements and transformations. The number of elements per item varied between one and three; the number of transformations was between zero and three. They found that the solution time is a direct function of the number of elements and the number of transformations. This indicates that individuals decompose the patterns of an analogy item sequentially by isolating the constituent elements one by one, as well as by performing the transformations in a serial manner. It also shows that not the number of elements, but only the number of transformations influences the percentage of errors. Mulholland (1980) concluded that with an increasing number of elements and transformations, it becomes more difficult to keep all of the performed steps in working memory, whereby the number of required transformations contributes more to item difficulty than the number of basic elements involved. An individual difference in the ability to solve matrix analogy problems is then related to differences in working memory capacity.

Green and Kluever (2001) conducted a components analysis of item difficulty in Raven's Matrices. She first identified 15 item components that might contribute to item difficulty. They were (a) vertical or horizontal orientation versus other orientation (coded as zero-1), (b) symmetrical versus asymmetrical (coded as zero-1), (c) progression versus non-progression (coded as zero-1), (d) the number of dimensions in the pattern (coded as zero-3), (e) straight lines versus curved lines (coded as zero-1), (f) the number of lines or solids (coded as zero-1), (g) the density of design (coded as zero-1), and (h) color versus black and white (coded as zero-1). Based on these characteristics, 60 items were developed. Regression analysis was carried out with item difficulties as the dependent variable and all of the 15 item characteristics were entered into a regression equation as both forced entry and stepwise entry. We can see that most of the 15 characteristics are figural characteristics. Another item difficulty was predicted based on the four characteristics that were identified as significant predictors of item difficulty. The multiple R^2 was .69. However, there are some limitations to this research. For example, this component analysis does not describe any elementary mental processes that may be necessary for problem solutions; only very obvious and observable features have been included in the analysis. However, the analysis of figural characteristics has provided some information which can be used in test design and in item difficulty judgment. The regression model used to predict new item difficulties has activated our concern that using a regression model based on existing data to predict new item difficulties can only provide fairly straightforward predictions.

Primi (2002) in his study synthesized four main factors from the literature (a) an increase in the number of figures, (b) the perceptual complexity of stimuli, (c) the complexity of the rules, and (d) an increase in the number of rules relating these figures. The main purpose of Primi's study was to identify the relative importance of the factors listed above. By manipulating these four sources of complexity, the author created two matrix tests to study the relative importance of these factors and their significant effect on item complexity. Using ANOVA and regression analysis methods, the author identified perceptual organization and the amount of information as the two variables which contributed significantly to an increase in item complexity. Furthermore, perceptual organization is the most important element, explaining 53.4% of the variance in item complexity.

If the number of figures and the number of rules relating these figures are grouped as one factor, we can see that there are three factors affecting the Matrix Inductive Reasoning item difficulties. Named by Primi (2002), these three factors are (a) Amount of Information Number of Elements and Rules, (b) the Nature of Relationships-type of rules, and (c) Perceptual Organization.

Amount of information includes the number of attributes and the number of rules involved in each figure; this is related to working memory capacity. When solving a matrix problem, one needs to keep in mind how many elements there are in the figure and what their relationships are. The more elements and rules the figure has, the larger working memory capacity will be needed.

Rule type is another source which affects item difficulties. Easier rules such as *constants in a row (column)* and a *distribution of 3* can be easily identified with perceptual identification. However, for the more difficult rules such as *quantitative pair-wise progression* and *figure addition*, subjects need mental or conceptual operation in addition to visual perceptual identification. Among these rules, the first six in Table 1 have received more attention than the others. However, the rule of *distribution of two values* and the rule of *figure addition/subtraction* are in fact the same. As we can see in the examples of these two rules, they both have two same values and a third non-value. Therefore, in this research, we consider the distribution of two values and the figure addition/subtraction rules as one: *the distribution of two plus zero*. Studies on the difficulty of the rule found that the order of the five rules from easiest to hardest is constant in a row (column), quantitative pair-wise progression, figure addition, distribution of 3, and distribution of 2 plus 1 (Carpenter et al., 1990). These rules are used in the Raven's progressive matrices. In this research, we have also used these five well-studied rules to design and develop Matrix Reasoning Tests.

Perceptual organization refers to how the figure is organized. The spatial order of elements can include proximity, similarity, continuity, and with common region (Mack, Tang, Tuman, & Rock, 1992), which add to the effect of the figure overlay distortion and fusion (Embretson, 1998). If one object is on the top of another, the drawing feature is called *overlay*; if two objects are put side by side with a common region in the same array location, the drawing feature is called *fusion*; if an ordinary shape is perceptually altered, bended, twisted, or stretched, etc., the drawing feature is called *distortion*. These features will distort the clues and make the items more difficult.

By balancing the three sources of difficulty, Matrix Reasoning Tests with range of difficulty distribution may be easily designed and developed.

Constructs of Matrix Inductive Reasoning

Dimensionality studies on Matrix Inductive Reasoning tasks are mostly conducted on Raven's Progressive Matrices tests (RPM), which use matrix problems to measure a person's ability to form perceptual relations and to reason by analogy independent of language and formal schooling. These tests are used widely to measure a person's intellectual ability in many studies and applied settings. The Standard Progressive Matrices (SPM) was the first series of tests developed for adolescents. It includes five sets, with a total of 60 items ordered from easy to difficult. The Colored Progressive Matrices (CPM) reformatted SPM into colors; this series is used for young children and special groups. The Advanced Progressive Matrices (APM) is a more difficult version of SPM. It is used for above-average adolescents and adults.

Although the Raven's test was designed to be a pure measure of *g* and was accepted as such by Spearman (1946) and Burke (1958), the contention of unidimensional nature of ability measured by either the SPM or APM has been challenged by other researchers (Dillion, 1981; Gustaffon, 1984, 1988).

Emmett (1949) conducted a factor analysis on the 60 items based on data collected from a sample of 11 year old children. Results show that SPM is a pure measure of *g*. Jensen (1998) has contended that "the total variance of Raven scores in fact comprises virtually nothing besides *g* and random measurement error" (p. 135). Raven, Raven and Court (2000) state that "The Progressive Matrices has been described as one of the purest and best measures of *g* or general intellectual functioning" (p. 34).

However, different results have been generated by other scholars (Adcock, 1948; Banks, 1949; Gabriel, 1954) who have contended that besides *g*, the Progressive

Matrices also measure a small factor of Visualization or Space. Gustaffson (1984, 1988) concludes that SPM contains a reasoning factor and a figural related cognition factor. Hertzog and Carter (1988) insist that SPM contains two factors: Verbal Intelligence and Spatial Visualization. Van der Ven and Ellis (2000) hold that SPM contains two significant factors which they identify as Gestalt Continuation and Analogical Reasoning. Lynn, Allik, and Irwing (2004) find that the three-factor solution for SPM can get the best fit of the data by using both exploratory factor analysis and confirmatory analysis method. The three factors are Gestalt Continuation, Verbal-Analytic reasoning, and Visuospatial Ability.

The same conflicting conclusions have been shown in the studies of APM. Alderton and Larson (1990) and Arthur and Woehr (1993) have claimed that a single-factor solution seems to be the best representation of the APM's structure, which means that APM is solely a measure of *g*. However, Dillon, Pohlmann, and Lohman (1981) have identified two factors in their study of APM; they named the two factors Pattern Addition/Subtraction and Detection of Pattern Progression. When Lim (1994) studied gender differences in performing APM, he concluded that APM is a pure measure of reasoning ability for boys, but that it also measures spatial ability for girls. Deshon, Chan, and Weissbein (1995) found two factors that they identified as Verbal-Analytic and Visuospatial abilities. Colom and Garcia-Lopez (2002) also conclude that the APM test measures both reasoning and spatial abilities.

The above studies on factors of the Raven's test have used factor analysis statistical techniques. The factors from the results of factor analysis are items clustered in groups according to their item difficulty. Difficulty factors are produced by the distributional properties of the dichotomous scores, but not the additional relevant cognitive factors that influence performance on the items (Rost & Gebert, 1980). The identified factors can be explained as characteristics of a group of items with the same level of difficulty but not the ability factors that the test measured. When the question of "what ability does the test measure?" is asked, it is not enough to simply conduct factor analysis on student item scores.

Problem Solving Process Analysis of Matrix Inductive Reasoning

Information processing analysis can aid in understanding the mental process of problem solving. Such analysis can tell what abilities accounts for the individual difference in problem solving and what item characteristics accounts for the different difficulties of item difficulty.

Researchers investigating the problem solving process of Matrix Inductive Reasoning tasks have focused primarily on the study of Raven's Progressive Matrices Test (Carpenter et al., 1990; Embretson, 1995; Hunt, 1974; Jacobs & Vandeventer, 1972).

Carpenter et al. (1990) used a variety of methods to analyze the cognitive processes used in solving problems presented in the Raven Progressive Matrices Test. Based on the detailed performance characteristics of verbal protocols, eye-fixation patterns, and errors, they describe the process as follows: In the first row, people encode and compare the figures with other entries to find corresponding figure parts. Next, they find that patterns emerge as rules from the pairwise comparisons as rules. People induce the rules one at a time until sufficient rules that account for all the variation among the entries in the first row have been found. A similar process occurs in the second row; in addition, there is a need to map the counterparts between the first row and the second row.

These rules are stored in the memory in a generalized form. The discovered rules are then applied to the third row to generate the missing entry.

Beginning with a task analysis of Raven's Progressive Matrices Test, Carpenter et al. (1990) found that five different types of rules govern the variation of the entries. These rules are always interchangeable in the same task. For a single rule, the difficulty order of the five types is (a) *constant in a row* where the same value occurs throughout a row but changes down a column, (b) *quantitative pairwise progression* where a quantitative increment or decrement occurs between adjacent entries in an attribute such as size, position, or number, (c) *figure addition or subtraction* where a figure from one column is added to (juxtaposed or superimposed) or subtracted from another figure to produce the third, (d) *distribution of three values* where three values from a categorical attribute are distributed through a row, and (e) *distribution of two values* where two values from a categorical attribute are distributed through a row; the third value is null.

If a problem involves multiple rules, subjects use a correspondence finding method to discover which elements in three entries in a row are governed by the same rule. Since cues for finding rules are ambiguous in some of the Raven's problems which are constructed by conjoining figures governed by several rules, the correspondence finding process is thus a source of difficulty.

Carpenter et al. (1990) furthermore claimed that Raven's item difficulty also varies with the number of rules. However, a large number of rules do not have a large effect on the process of inducing rules. Instead, the number of rules affects the goal-management processes that are required to construct, execute, and maintain a mental plan of action during the solution of the multiple rule problems. Carpenter et al. used two experiments to test this hypothesis. The purpose of Experiment 1 was to reveal the process and the content of thought when subjects were solving each Raven problem. Think aloud and eye fixations methods were applied to some subjects. Other subjects were asked to work silently and then describe the rules that stimulated their response. The results from both groups showed the incremental nature of the processing: the subjects solved a problem by decomposing it into successively smaller sub-problems and then proceeded to solve each sub-problem one at a time.

Based on this result, the authors put forward the other hypothesis that a major source of individual differences "is the ability to generate sub-goals in working memory, to monitor progress toward attaining them, and to set new sub-goals as others are attained" (p. 413). The whole process of generating sub-goals, monitoring progress, and setting up new goals is called goal management. In Experiment 2, subjects were first administered the Raven Progressive Matrices Test; they were then trained with the goal-recursion strategy to solve the Tower of Hanoi puzzle task, a cognitive task involving extensive goal management. Significant correlation between the two tasks leads to the conclusion that a major source of individual difference in the Raven test derives from the generation and maintenance of goals in working memory.

To specify the process required to solve the Raven problems, two simulation programs were developed: FAIRAVEN performed at the level of the median college student in the sample, and BETTERAVEN performed at the level of the best subjects in the sample. These two models verified the results of Experiments 1 and 2. The authors conclude that "what one intelligence test measures, according to the current theory, is the common ability to decompose problems into manageable segments and iterate through them, the differential ability to manage the hierarchy of goals and sub-goals generated by this decomposition, and the differential ability to form higher level abstractions" (p. 429).

Another investigation on the information process of matrix problem solving was conducted by Embretson (1995). Her study examined student performance on 150 matrix items generated based on a cognitive theory of abstract inductive reasoning. The goal of Embretson's study was to estimate the relative contributions of individual differences in general control processing and in working memory capacity to individual differences in performance on these matrix items. Embretson (1995) attempted to distinguish the relative importance of executive functions (Belmont & Butterfield, 1990) and the role of working memory (Carpenter et al., 1990) by using a multi-component latent-trait model. Two latent variables which were responsible for individual differences in the task were posited: working memory capacity and control processing results showed that control process latent variable accounting for more variance than the working memory latent variable.

While control processes played an important role for high levels of performance on difficult reasoning tasks, Embretson's (1995) also found that working memory or attention resources also play important roles.

In conclusion, goal management process and working memory are important aspects of the process of solving Matrix Reasoning problems.

Working Memory and Inductive Reasoning

The important role of working memory in solving inductive reasoning tasks has been strongly claimed by many researchers (Carpenter et al., 1990; Marshalek et al. 1983). Buehner, Krumm, and Pick (2005) even proposed that reasoning is equal to working memory. In this section, we will review studies on the relationship between working memory and inductive reasoning.

Although it was first referred to as *short-term memory*, scholars now emphasize the manipulative function of working memory. Researchers have proposed many different models to explain the structure and function of working memory. Of them, Baddeley's (1992) model was taken as the most important influential one. Baddeley's (1992) model included three elements: (a) the visuospatial sketch pad which is a visuospatial storage system, (b) the phonological loop which stores verbal based information, and (c) the central executive system which is an attention-controlling system and coordinator for the two storage components and their interactions. Working memory is an important indicator of reasoning ability.

Kyllonen and Christal (1990) designed some working memory tasks specifically used to measure Baddely's concept of working memory. They found structural coefficients of .80 through .88 in four large studies between working memory and reasoning ability. Although Keyllonen's et al. (1990) work was criticized that some of the tasks for working memory test and reasoning test are the same thus increased the correlation, the overall high correlation coefficients showed the strong relationship between working memory and reasoning ability. Kyllonen and Christal (1990) argue that all reliable variation in reasoning can be explained by limitations on working memory capacity.

Conflicting results existed in other research. These researches emphasized the important function of executive attention. Researchers argued that the shared variance among measures of working memory span and complex cognition reflects primarily dues to the contribution of executive function, rather than specific storage capacity (Engle & Kane, 2004). As Kane, Hambrick, et al. (2004) pointed out,

> correlations between WM [Working Memory] span and complex cognition are jointly determined by general executive-attention and domain-specific storage but primarily by executive attention. Thus, a WMC [Working Memory Capacity] measure should be quite general in predicting cognitive function. That is, the memory span test could be embedded in a secondary processing task that is unrelated to any particular skill or ability and still predict success in a higher level task. Evidence supporting this view comes from three sources: (a) manipulating the processing demands of verbal WM [Working Memory] pan tasks and noting their relations to comprehension, (b) examining the between verbal WM span and measures of general fluid intelligence, and (c) examining the link between verbal WM span and low-level attention capabilities. (p. 190)

Relationship between Raven's Advanced Progressive Matrices in conjunction with working memory was also examined by some researches. Jurden (1995) reported the correlation of WM performance to the Raven reading span and computation span of .20 and .43 respectively. Babcock (1994) found a higher relationship of .55 between working memory and Raven performance. Although these studies differed in the degree of relationship reported, they both agree that working memory is positively correlated with performance on the Raven's test.

As a result, the author concluded that working memory should be tested by using multiple facets. When solving Matrix Reasoning problems, people need to store the identified rules governing the corresponding figure parts in their working memory. This memory is related to the observed patterns and shapes. The goal management process is also needed to control what information should be stored in the working memory and what information should be released from memory. Considering that Matrix Reasoning tasks are composed of figures, we chose two types of working memory tests that are similar in regard to the memory functions discussed above. The two tests include the Binary Number Working Memory Test (BNWMT) and the Shape Memory Test (SMT). In the BNWMT, each item is a number containing a series of ones and zeroes such as 110110110 or 01100110. The task of the examinee is to identify and remember the pattern of the digits. In the SMT, the memory tasks are a number of shapes that examinees are expected to remember.

Above is a review of research related to the domain of matrix inductive reasoning. These studies have provided the fundamental theories and raised questions for the further research.

Chapter 3: Method

Chapter 3 discusses the methods used to conduct this research. It introduces how the instrument was developed to measure the matrix reasoning abilities and the subabilities. The process of obtaining samples and collecting data are also elaborated. Procedures for analysis the data is then laid out. The chapter also includes the methods used to address each research question.

Instrument Development

Five tests were developed to measure the subablities identified from the previous discussion. Each of them is described in detail below.

Matrix Reasoning Test

A 16-item Matrix Reasoning Test was constructed to measure the matrix inductive reasoning ability. It was patterned after Raven's series of tests. The format and content of this new test is described in the following sections.

Item format. Each matrix item consisted of nine entries arranged in three rows and three columns. The last entry on the lower right contained a question mark; all other squares were figures. There were six answer options for each matrix item. Subjects were asked to choose one correct answer from the six options to complete the blank entry. An example of such a matrix item is shown in Figure 1.

Item content specifications. The matrix items were constructed by varying the four aspects which affected the difficulty of the items (a) the number of elements, (b) number of rules, (c) rule types, and (d) figural complexity. The five rules used in the tests included (a) constant in a row (column), (b) distribution of two values plus zero, (c) distribution of two values plus one, (d) quantitative progression, and (e) distribution of three. The constant in a row rule was that the same attribute occurs throughout a row (or a column); the rule of distribution of two values plus zero was that two identical values are distributed through a row while the third value is none; the rule of distribution of two values plus one was that three values from a category attribute distributed through a row with two of the three values are identical while the third one was different from the other two; the quantitative progression rule was that a quantitative increment or decrement of a value occurs between the two adjacent entries. To further understand these rules in the items, refer to Table 2 for content specification. In this table, rules were listed in the first row while items were listed in the first column. The number 1 in the cross cells means that the rule in this column has been used once in the item. If more than one number 1 appears in the cell, it shows that the rule in this column has been used more than once.

Rule Induction Test

The purpose of the Rule Induction Test was to determine how well the examinees discover or recognize a particular rule. Without the interaction of other factors such as complex figural or multiple elements, could the subjects figure out what the rule was?

At the top of the page, two rows of simple figures were given to the subjects. Figures in these two rows were governed by the same rule. Four options were listed after the instruction. One or more of the options shared the same rule as the previous two. The subjects were asked to choose the one or more options which shared the same rule as the previous two rows. Refer to Figure 2 for an example of the Rule Induction task. In this test, each items used a different single rule.

Table 2.

Rule Types and Number of Rules Used in Matrix Reasoning Test

Rule Type*	Constant	2+0	2+1	Quantitative	D3	Rules in Each Item
1				1		1
2					1	1
3	1					1
4		2				2
5	1				1	2
6			1		1	2
7	1	2				3
8	1				2	3
9		2				2
11	1				1	2
12					2	2
13	1	1	1			3
14			1	2	2	4
13		3				3
16		3				3

**constant: constant in a row (or column); 2+0: distribution of two plus zero (figure addition); 2+1: distribution of two plus 1. Quantitative: quantitative progression.D3: distribution of three.*

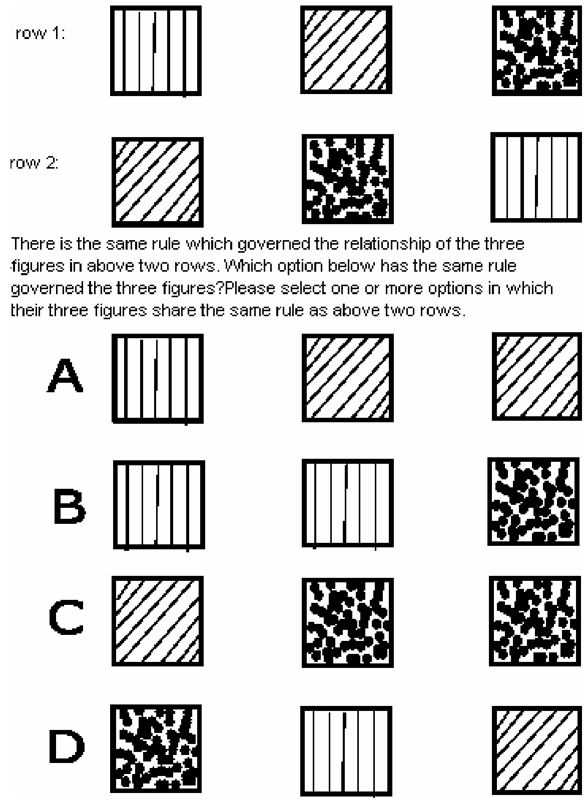


Figure 2. Example for a Rule Induction Test item.

Rule Application Test

The Rule Application Test was used to examine subjects' ability to apply a rule to a new situation. In this test, the researcher first decomposed the figures. A clear rule description for a particular part of the figure was then was provided. Two figures were given; the subjects were asked to find out what the third one should be by applying the rules provided. Figure 3 is an example of a Rule Application item.

Table 3 presents the rule types and the number of rules used in each item of the Rule Application Test.

Working Memory Test

This test focused on working memory capacity. It included two parts: the BNWMT and the SWMT.

Binary Number Working Memory Test. This test asked subjects to remember several binary numbers ranging from 3 to 12 digits. The ratio of number length/display time was 1:1. For example, for a 3-digit number, the display time is 3 seconds; for a 4-digit number, the display time is 4 seconds, and so on. The binary number was shown in the first page, then automatically went to the answer page after the display time expired. In the answer page, the subjects needed to type in the number they saw on the previous page. There were 10 items. One example of an 8-digit binary number was 10010101.

Shape Working Memory Test. SWMT asked subjects to remember the shapes they saw on a previous page. These shapes were regular geometric shapes. The number of different shapes in each item ranged from three to nine. The shapes were displayed for a fixed time interval; then an answer page was displayed. The subjects were asked to

rule1: For the outside part: two shapes are the same,

the third one is none.

rule2: For the inside part: two shapes are the same,

the third one is none.

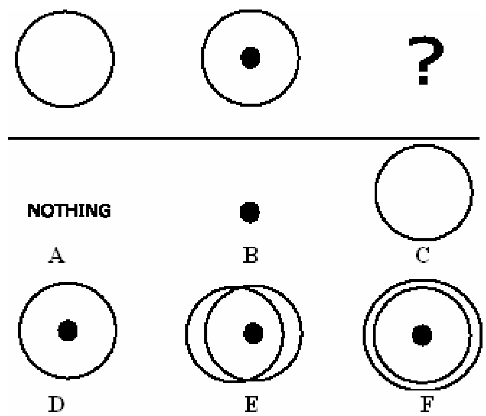


Figure 3. Example of a Rule Application Test item.

Table 3.

Rule Types and Numbers Used for Each Item in Rule Application Test

Rule Type*	Constant	2+0	2+1	Quantitative	D3
1				1	
2				1	1
3	Bad item				1
4		2			
5	1	1			
6		1			
7		3			
8					3
9		1			1
10			1		1
11	1	1			1

** constant: constant in a row (or column); 2+0: distribution of two plus zero (figure addition); 2+1: distribution of two plus 1. Quantitative: quantitative progression.D3: distribution of three.*

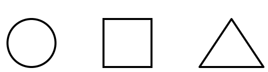
choose figures they saw from a list of 20 shape options. The ratio between the numbers of shapes and the display time was 1:1.5. The following is an example of a three SWMT item: 

Figure Detection Test

The Figure Detection Test was used to measure figural decomposition ability. For complex figures in the Matrix Reasoning tasks, subjects were asked to decompose them to independent parts and find the correspondence rule among the figures. The Figure Detection Test used the format of Hidden Figure tasks. In this test, subjects were asked to find a given shape which was embedded in a complex one. Figure 4 shows an example of a Figure Detection item that was used in this research.

Sampling

Structural equation modeling is sensitive to sample size and requires relatively large sample sizes. The sample should consist of a minimum of 100 subjects and should be at least five times larger that the number of variables being analyzed (Bantler & Chou, 1987). Because the number of test items developed in this research was 52, at least 260 subjects should be included according to the minimal sample size rule.

Students from grade 6 through grade 8 participated in this study. These students were from Beijing and Shanghai. Items in this research were developed from Colored Progressive Matrices and the Advanced Progressive Matrices, being more difficult than the Colored Progressive Matrices, yet easier than the Advanced Progressive Matrices. According to Raven's test, Colored Progressed Matrices are used with younger children And special groups and the Advanced Progressive Matrices are used with above teachers, and students. The four principals, ten teachers, and each student in the selected classes who agreed to participate in the research were asked to sign an agreement form.

Data Collection

To collect the data, a database was developed by using the PHP and MySql computer languages. The five tests were then connected to the database. The database was stored in the computer lab servers in the two participating schools in Beijing and

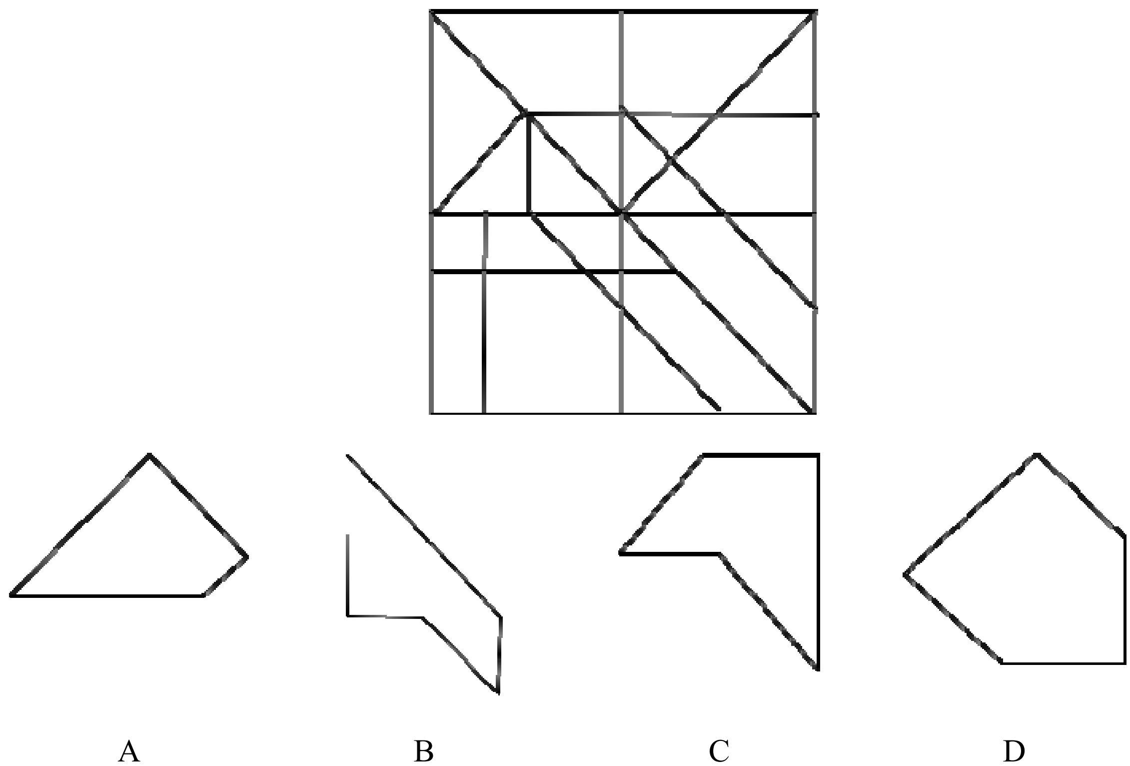


Figure 4. Example of a Figure Detection Test item.

Shanghai. The five tests were administered to whole group of students over the internet on each campus to one classroom at a time. Trained graduate students were in the computer lab on each campus to assist the students in completing the tests. No time limit was imposed. Students completed the test during the last class period of the school day and were able to respond to each test at their own pace. All the collected data were stored on the computer lab servers of participating schools.

Pilot Study Data Collection

A pilot study was conducted with a small group of students before the tests were administered to the full sample of students. The purpose of this pilot study was to examine item characteristics. Further actions including modifying, deleting, and adding items to the test were adopted based on pilot study analysis results. One hundred and eleven students from grade 7 participated in the pilot study. Of the 111 students, 53 (47.7%) were female, 54 (48.6%) were male. Four students (4.3%) did not report their gender. Seventy eight (70.3%) of the 114 students were age 12, 27 (24.3%) were age 13, and 6 (5.4%) students did not report their age.

Items were deleted and added based on the item difficulties from the pilot study. Table 4 illustrates the item changes for each individual test.

Formal Data Collection

The revised tests were administrated to a sample of 352 students in China from grades 6 to 8. Students who participated in the pilot study were not included in the data collection. The same data collection procedure that was used in the pilot study was adopted for use in the general sample data collection. Table 5 shows student participation numbers and rates by grade and gender.

Table 4.

Item Changes in the New Test

Tests	Number of Items Deleted	Number of Items Added	Number of Items in the New Test
Binary Number Working Memory Test	4	2	5
Shape Working Memory Test	1	0	4
Figure Detection Test	4	0	4
Rule Induction Test	0	0	6
Rule Application Test	0	0	12
Matrix Reasoning Test	2	0	14

Upon receipt, the data was inputted into Microsoft Excel. Under the supervision of a graduate student, the inputted data was carefully checked for errors to ensure accuracy. After the data was considered clean, analyses were conducted.

Data Analysis

Scoring and Data Cleaning

Through the use of Microsoft Excel logical functions, the initial answers of students were scored with either a 1 (correct) or a 0 (incorrect) and saved in a different data file. In the data collection process, instances of missing data were encountered. If any of the items in one or more of the five tests were not completed, the case was directly eliminated from the analysis. Of the 352 participants, 18 cases were eliminated. The analysis of this study is based upon the remaining 334 complete effective cases.

Table 5.

Student Participation Numbers and Rates by Grade Enrollment and Gender

Data input

Grade	Number of participants	Gender		
		Males	Females	Unkown
6	117 (35.03%)	49 (14.67%)	65 (19.46%)	3 (.90%)
7	116 (34.73%)	52 (15.57%)	58 (17.37%)	6 (1.80%)
8	100 (29.94%)	39 (11.68%)	59 (17.66%)	2 (.60%)
Null	1 (.30%)	0 (0.00%)	0 (0.00%)	1 (.30%)
Total	334	140 (41.92%)	182 (54.49%)	12 (3.59 %)

Software Selection

Many software packages were available for the modeling analysis. The most commonly used are AMOS (Arbuckle, 2003; Arbuckle & Wothke, 1999; SPSS, Inc., 2005), LISREL (Jöreskog & Sörbom, 1996; Jöreskog, Sörbom, Du Toit, & Du Toit, 2001), and MPLUS (Muthen & Muthen, 2001). Of these software packages, MPLUS is the most well known for its capacity to deal with complicated models, handling both continuous and categorical data. Using proper estimation methods such as analyzing tetrachoric correlations and using a robust unweighted or weighted least-squares estimator, MPLUS can conduct both EFA and CFA with dichotomous data. Since the data used in this research is dichotomous, MPLUS was used in this research.

Reliability and Validity

Cronbach's alpha coefficient was computed to estimate the internal consistency reliability of the scores obtained from each test. Evidence of convergent validity was obtained by examining the factor coefficients (Anderson & Gerbing, 1988). Convergent validity is demonstrated if the items which are associated have significant high coefficients (greater than twice its standard error) on the same factor and if the factor loading is relatively high (greater than .06). Evidence of discriminant validity was confirmed by showing that the confidence intervals (± two standard errors) around the estimated correlation coefficients for a given pair of factors contained the value of 1.0.

Measurement of Construct for each Test

To explore the constructs of the developed tests, CFA models based on related literature and theories were tested. Three models were specified for each test (a) a unidimensional model in which all of the items using different type and number of rules and figures were represented by a single factor; (b) a first-order oblique model that included separate factors for items in different difficulty levels; and (c) a second-order factor model used to account for covariation among factors. For each test, the unidimensional model was first tested. Only if this unidimensional model did not fit the data acceptably would the other two models be tested. Otherwise, this unidimensional model would be applied to the following structural equation modeling analysis.

The CFA analysis was conducted using MPLUS. As mentioned in the preliminary research by Muthén, DuToit, and Spisic (1997), an optimal estimation method for categorical outcomes was weighted least-squares with mean and variance adjustment

(WLSMV). Since the data in this research was dichotomous, the WLSMV estimation method was the best choice.

To evaluate the fit of each model, a combination of criteria consisting of chi-square tests and several descriptive fit indices were adopted. The chi-square test is a basic statistical procedure to test for model fit. A nonsignificant chi-square value indicates good model fit. The chi-square test is sensitive to sample size, being particularly hard to obtain with a large sample size. Besides the chi-square test, MPLUS also provided the fit indices of the comparative fit index (CFI), the Tucker-Lewis index (TLI), the Standardized Root-Mean-Square Residual (SRMR), the Weighted Root Mean Square Residual (WRMR), and the Root Mean Square Error of Approximation (RMSEA). Of these indices, SRMR is not recommended for dichotomous data (Yu, 2002). Therefore, CFI, TLI, RMSEA, and WRMR were chosen for the construct analysis for each individual test. For the model test which using continuous data, SRMR is adopted instead of WRMR. CFI measured the improvement of the fit by comparing the hypothesized model with a null model in which the measured variables were assumed to be unrelated (Bentler & Bonett, 1980). Hu and Bentler (1999) recommended a CFI cutoff value of around .95. RMSEA shows the average difference between observed and expected covariance; a RMSEA value below .05 indicates a good model fit (Browne & Cudeck, 1993).WRMR was proposed by Muthén and Muthén (1998-2001) as being suitable with non-normal outcomes. A model is accepted when the WRMR value is equal to or less than 1.0 for the model (Yu, 2002). Cut off values equal or less than .08 was recommend by Yu (2002) in her research. In summary, joint criteria for a good fit of indices according to this study are CFI $\geq$ .95, RMSEA $\leq$ .05, and WRMR $\leq$ 1.00 or SRMR $\leq$ .08.

Research Questions and Methods

Research questions and the corresponding methodologies are addressed in this section.

Research Question 1: Which of the Alternative Models is the Most Valid Representation of the Domain of Matrix Reasoning Problem Solving?

In line with the three theoretical hypotheses regarding the domain of Matrix Reasoning problem solving referred to earlier, three alternative models (Figures 5 to 7) were defined. The common part of the three models was that the four predictor variables, working memory, figural decomposition, rule induction, and rule application, had direct effects on matrix reasoning.

The first model (Figure 5) is a component model. In this model, in addition to its direct effect on matrix reasoning, working memory also directly affects figure detection, rule induction, and rule application. Each of these three variables also predicts matrix reasoning.

All other variables in the second model (Figure 6) directly affect the matrix reasoning variable. Also, working memory, rule induction, and figure detection indirectly affect matrix reasoning through the mediator variable rule application. There are no relationships specified among the latent variables figure detection, working memory, and rule induction.

The third model (Figure 7) tests the problem solving process theory. Working memory directly affects all other variables. Simultaneously, a step-by-step process is required from figure detection to rule induction, rule induction to rule application, and finally from rule application to matrix reasoning.

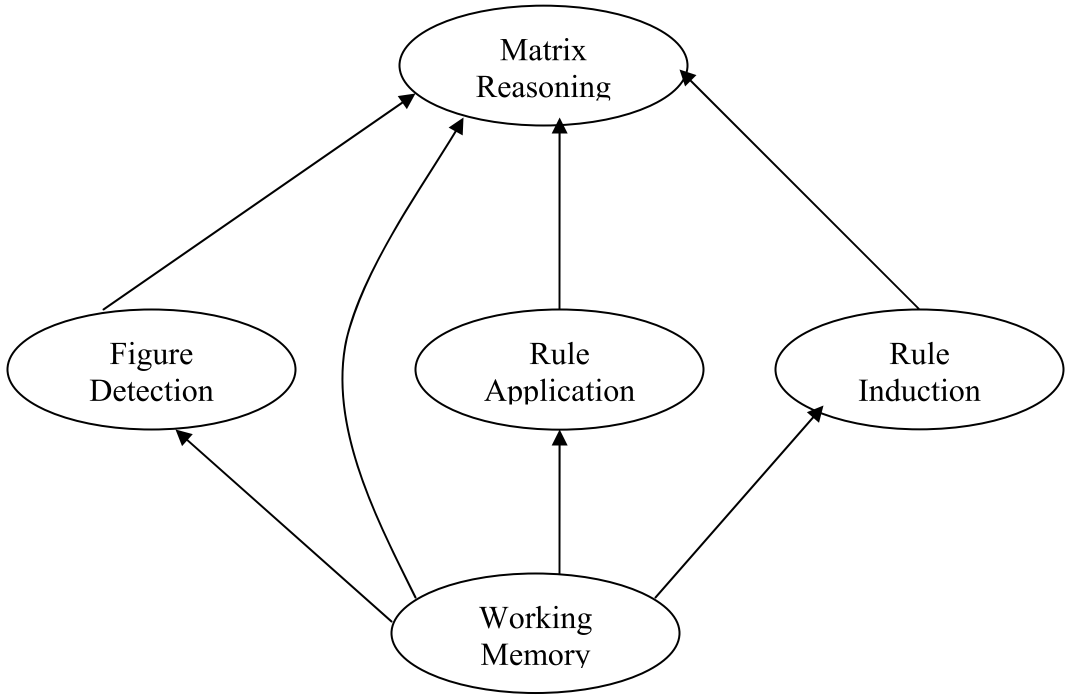


Figure 5. Hypothesized model 1: component model.

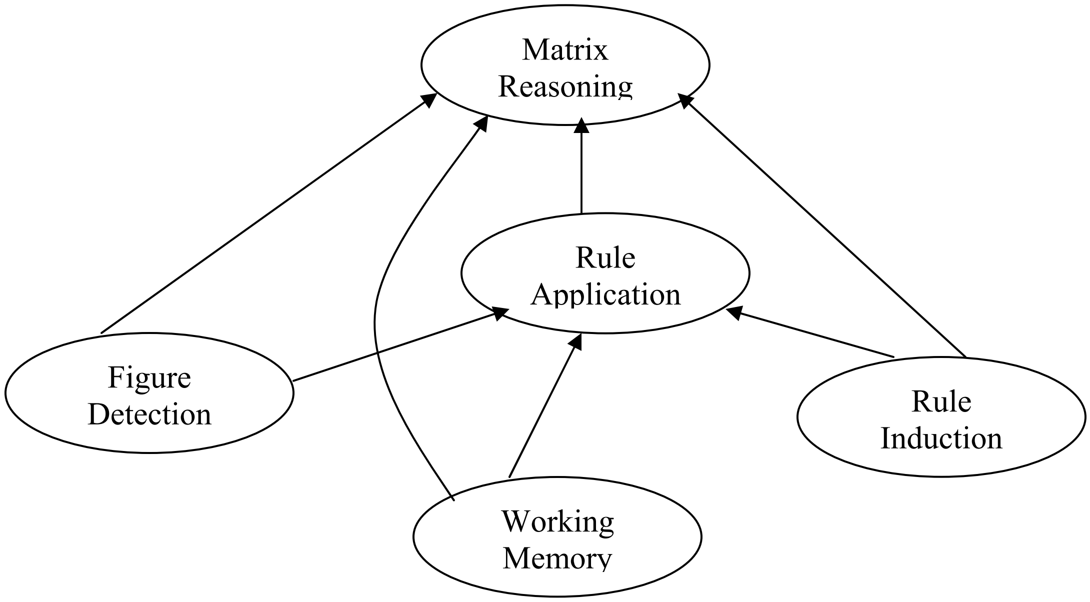


Figure 6. Hypothesized model 2: rule application as mediator model.

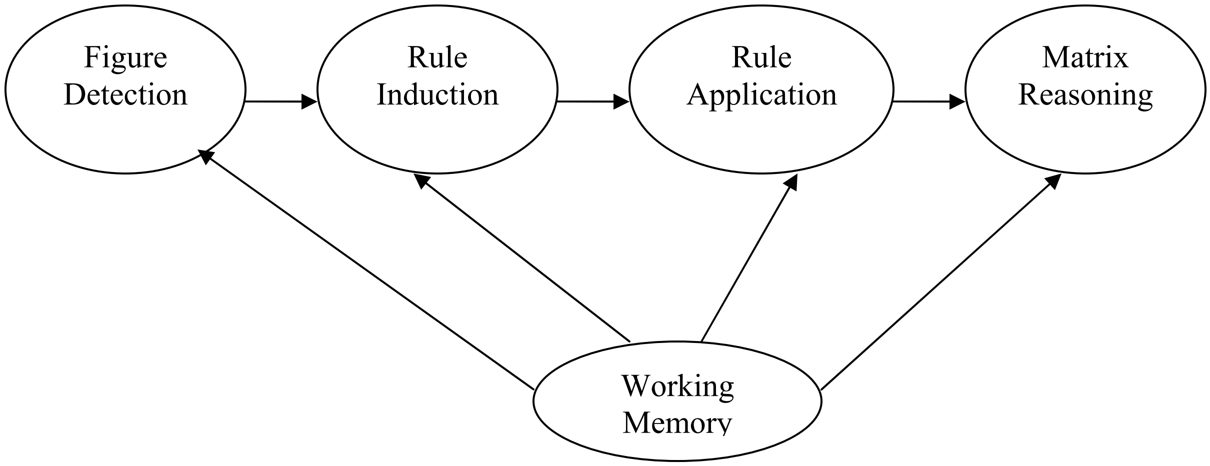


Figure 7. Hypothesized model 3: problem solving process model.

The three models were combined into a full model in which all latent variables were intercorrelated with each other. The model which fits the actual data with the greatest degree of accuracy would be preferred.

Data preparation for the model analysis. Structural equation modeling (SEM) was used to evaluate the three hypothesized models. SEM is a family of statistical techniques that includes path analysis and factor analysis. It can be used to test the plausibility of hypothesized patterns of causal effects involving latent variables. There are two ways to prepare the data for SEM analysis. One way is to export the chosen constructs with the original items to the model. The second way is to create scales by adding up item scores under the same factor. For tests with unidimensional construct, a parcel was used to create two or more observed variables. Because the second method is parsimonious and has fewer errors, the second method was adopted.

Procedure to test the models. To test the structural equation models, a two-step modeling method introduced by Anderson and Gerbing (1988) was adopted.

The first step is to respecify the structural equation models to a CFA measurement model. Testing the validity of the measurement model before evaluating the structural model allows the research to distinguish rejections of the proposed model because of problems stemming from measurement inadequacies from problems related to the actual proposed theory (Mueller, 1996). Since all SEM models share the same latent variables, only one CFA model will be addressed. This measurement model expresses all possible associations among the latent variables. It considers each observed indicator as a linear combination of a latent unobserved factor plus random measurement error. Testing the validity of the measurement model before evaluating the structural model allows the research to identify problems stemming from measurement inadequacies (Mueller, 1996). Since the variables in this model are continuous variables (scales), the Maximum Likelihood (ML) estimation method was used for the model testing.

Given an acceptable CFA model, the second step is to test and compare the proposed SEM models. The three SEM models were tested under the same available data as used in the first step. Results from the MPLUS software for the three models were then compared to the newly identified CFA model. As the method applied in the measurement model analysis, Chi-square values and joint criteria for a good fit of indices of CFI & TLI $\geq .95$, RMSEA $\leq .05$, and WRMR ≤ 1.0 were obtained to evaluate whether the hypothesized structural model is a valid representation of the domain constructs.

Research Question 2: What modifications can be made to improve the preferred SEM model?

The model preferred in question 1 was theoretically specified. The MPLUS program also provides a MODINDICES index which suggests empirical modifications to improve the best model fit based on the actual data. The MODINDICES indices were examined to see whether some reasonable suggestions could be made to the model in question 1. If any modifications could be adopted, new modified models should be retested.

Research Question 3: What Are the Significant Direct and Indirect Effects of the Latent Variables?

Direct and indirect effects can be specified using the MPLUS program. All path coefficients from the final model in question 2 were examined. These coefficients were reported from MPLUS results. The outputs from MPLUS included the Estimates (the model-estimated values for each parameter), the standard errors of the parameter estimates, the values of the parameter estimates divided by the standard error, and two types of standardized parameter estimates. MPLUS outputs did not provide *p*-values for each estimated parameter. The ratio of the estimate divided by the standard error was used to indicate the significance of the parameters. The ratio statistical test is an approximately normally distributed quantity (z-score) in large samples. Values that exceed +1.96 or fall below -1.96 are significant below $p = .05$ for a two-tailed test. Conclusions were drawn based on these ratio values.

The relative strength of associations across latent variables can be reflected from standardized parameter estimates. Of the two types of standardized parameter estimates, the first type of coefficients are standardized using latent variable variance while the second type of coefficients use the latent variables as well as the observed variable variances. In this question, the two types of coefficients were the same because the parameter estimates involved only latent variables.

Chapter 4: Results

Chapter 4 presents the findings of the study based on the data analysis. In this chapter, the descriptive statistics for each test used in this study are reported followed by detailed analysis on each of the research questions.

Test Score Reliability

Item internal consistency reliability estimates were calculated using SPSS. The Cronbach's alpha for each of the five tests is reported in Table 6. The Figure Detection test had the lowest alpha value of .53. The alpha coefficient of .62 for the shape Working Memory test was moderately low too. One reason for the low reliability estimates for these two tests is that each of them included only four items. The lower reliability estimates for these two tests indicate that interpretations about examinees based on scores from these tests should be made with caution. In general, the Cronbach's alpha in other tests indicated acceptable internal consistency reliability because all the coefficients were above the recommend level of .70 (Nunnally & Bernstein, 1994).

Table 6.

Descriptive Statistics for Each Test

Test	Number of Items	Mean	Standard Deviation	Coefficient Alpha
Matrix Reasoning	14	.64	.07	.85
Rule Application	12	.51	.17	.88
Rule Induction	6	.81	.07	.89
Figure Detection	4	.50	.11	.53
Binary Working Memory	5	.76	.07	.70
Shape Working Memory	4	.41	.03	.62

Confirmatory Factor Analyses

Since multiple indicators were used to measure the constructs investigated in this research, analyses were required to demonstrate that the items within each test all measured the same trait. That is, each set of alternate indicators has only one underlying trait or construct in common (Hattie, 1985). This test of unidimensionality was accomplished by conducting a separate confirmatory factor analysis for each test

Construct Analysis for Matrix Reasoning

From literature reviews on the dimensionality of the Raven's tests that used matrix reasoning tasks, three different measurement models were proposed: (a) a unidimensional model in which all of the items were represented by a single factor, (b) a first-order three factors oblique model, and (c) a second-order factor model in which a higher order factor accounted for the covariation among the three first order factors.

The standardized pattern coefficients in the three substantive alternative models are reported in Table 7. The standardized factor loadings for the three models were significant at the .001 level. The significant loadings of the related variables on the same factor indicate a common construct and hence support the convergent validity of scores on the test.

Table 8 reports the correlation coefficients for each pair of factors based on the results of the three-factor oblique model. The estimated correlations between the factors ranged from .71 to .79. For all correlation coefficients, the confidence interval did not include the value of 1.0. This finding suggests acceptable discriminant validity for scores obtained from the Matrix Reasoning test.

Table 7.

Standardized Factor Coefficients on Matrix Reasoning Items for Each Identified Factor across Alternative Factor Models

Items	Unidimensional	Three-factor Oblique	Second order Hierarchical
Factor 1			
2	.51**	.58**	.58**
3	.74**	.84**	.84**
4	.53**	.61**	.61**
5	.71**	.80**	.80**
Factor 2			
7	.46**	.49**	.49**
8	.55**	.59**	.59**
9	.72**	.77**	.77**
11	.68**	.72**	.72**
12	.50**	.53**	.53**
Factor 3			
6	.59**	.64**	.64**
13	.36**	.40**	.40**
14	.26**	.30**	.30**
15	.70**	.78**	.78**
16	.47**	.52**	.52**

**$p < .001$

Table 8.

Estimates of Intercorrelations among Three Factors on Matrix Reasoning Tests

Factors	1	2	3
1	1		
2	.72 (.05)	1	
3	.71 (.05)	.79(.06)	1

Note. Figures in parentheses are standard errors. Example: 95% confidence interval of estimate correlation between factor 1and factor 2 is calculated as .72 ± 1.96 x (.05).

Chi-square tests and goodness of fit measures (CFI, RMSEA, and WRMR) for the three models as obtained from MPLUS 3.0 (Muthén & Muthén, 2003) are reported in Table 9. As can be seen in Table 9 the chi-square value of 64.34 with 51 degrees of freedom for the three factor oblique model and the second-order hierarchical model was not statistically significant (.13) at the .01 level. The chi-square test showed a significant result for the unidimensional model with a *p*-value less than .01. This suggests that while the three factor oblique model and the second-order hierarchical model fit the data acceptably, the unidimensional model does not. Corroborating evidence can be provided by using the goodness-of-fit index. Although the goodness-of-fit indices for all three of the models meet standard requirements (CFI > .95 and RMSEA < .05); however, when compared to the unidimensional model, the three factor oblique model and the second-order hierarchical model had higher CFI but lower RMSEA. We can conclude that the three-factor oblique model and the second-order hierarchical model each have better model fit.

Table 9.

Goodness-of-Fit Indices for Null and Alternative Factor Models of Matrix Reasoning

Factor Model	*df*	χ^2	*p*-value	CFI	RMSEA	WRMR
Baseline model	47	561.90	.00			
Unidimensional model	52	79.50	.009	.947	.040	.098
Three-factor oblique model	51	64.34	.133	.978	.026	.086
Second-order hierarchical model	51	64.34	.133	.978	.026	.086

The second-order hierarchical model appears to be equivalent to the oblique model due to identical chi-square test results and fit indices. This is most likely due to the fact that the two models involve the same number of parameters. However, the second-order model, which explains the high correlations among the lower-order factors, is more theoretically desirable as far as this research is concerned. The model also provides a good reason that there is a general higher order factor which is measured by all of the observed variables. This higher order factor may be the *g* factor that has been studied thoroughly by previous researchers.

Further exploration of the data shows that the items associated with each of the three factors are clustered in groups ordered by the item difficulties. Factor 1 consists of the easy items; Factor 2 is related to items of intermediate difficulty; and Factor 3 includes the most difficult items. Apparently, when these three factors are added to an oblique model, it is not able to provide a good interpretation of what the Matrix Reasoning items have measured. Therefore, the second-order model will be used in the SEM analysis in the rest of this study.

Construct Analysis for Rule Application Test

Rule Application items are different than Matrix Reasoning items in that the rule used in each figure item is stated in the item stem. The same three measurement models—the unidimensional model, the first-order two-factor oblique model, and the second-order factor model—were also tested for the Rule Application Test. Since there are only two first-order factors for the second order factor, this second order factor model was underidentified. The standardized factor coefficients of the two identified models are provided in Table 10. Since there are two or more items are associated with significant coefficients on the same factor, it reflects that these items are under a common construct (Anderson & Gerbing, 1988). The significant coefficients hence are evidence of convergent validity of scores on this test.

The intercorrelation estimate of the two factors based on the results of the two-factor oblique model is .80 with standard error of .04. The confidence interval for correlation value of .80 is from .72 to .88, which did not contain the value of 1.0.

Table 10.

Standardized Factor Coefficients on Rule Application Items for Each Identified Factor across Alternative Factor Models

Items	Unidimensional	Two-factor Oblique
Factor 1		
1	.37**	.38**
2	.76**	.77**
3	.48**	.48**
4	.76**	.76**
5	.44**	.45**
6	.80**	.80**
7	.78**	.79**
8	.92**	.93**
Factor 2		
9	.53**	.59**
10	.87**	.40**
11	.26**	.30**
12	.45**	.52**

**$p < .001$

Conclusion can be drawn that there is an acceptable discriminant validity of the Rule Application Test scores.

The model test results for the Rule Application Test are reported in Table 11. Although the model fit indices for CFI, RMSEA, and WRMR meet the standard requirement, the overall model test for the unidimensional model yielded a $\chi^2 = 65.14$ (N = 41), $p < .05$, indicating that this model generally fit poorly. The chi-square value of 47.04 with 40 degrees of freedom for the two-factor oblique model has a corresponding p-value of .21, suggesting that this model fits the overall data well. From a close-fit perspective, the fit indices CFI = .994, RMSEA = .023, and WRMR = .764 also meet standards. As discussed in the Matrix Reasoning model test, the items associated with each these two factors were clustered in terms of item difficulty.

Table 11.

Goodness-of-Fit Indices for Null and Alternative Factor Models of Rule Application Test

Factor Model	*df*	$\chi 2$	*p*-value	CFI	RMSEA	WRMR
Baseline	30	1199.01	.00			
Unidimensional model	41	65.14	.010	.979	.042	.895
Two-factor oblique model	40	47.04	.206	.994	.023	.764

Construct Analysis for Rule Induction Test

Single rule reasoning was used in the Rule Induction Test. Consequently, students were not required to decompose figures. Neither were they required to memorize reasoning facts. Therefore, the structure of this test was built first as only a unidimensional model. As shown in Table 12, the chi-square value of 10.93 which was evaluated with four degrees of freedom yielded a corresponding *p*-value of .28. This p-value is too high to reject the null of a good fit. Corroborating evidence is provided by CFI and RMSEA fit statistics. The fit indices for the three statistics are .995, .025 and .062, respectively which all meet the standard requirement. Generally speaking, the unidimensional model fits the data in an acceptable manner. Therefore, no further model was estimated for construct analysis of the Rule Induction Test.

Table 12.

Goodness-of-Fit Indices for Null and Alternative Factor Models of Rule Induction Test

Factor Model	*df*	χ^2	*p*-value	CFI	RMSEA	WRMR
Baseline model	15	397.13	.000			
Unidimensional model	9	10.93	.280	.995	.025	—

Standardized factor coefficients of the item loadings on the factor are presented in Table 13. All of the coefficients are significant at .01 levels, showing good convergent validity in the test scores.

Construct Analysis for Figure Detection Test

A Figure Detection Test was used to assess students' visual spatial ability. In this test, students were asked to segregate simple pictures embedded in a complex visual configuration. Since no argument was found in the literature about the dimensionality of Figure Detection Test, we hypothesized that this test is a unidimensional visual spatial ability test. The model test results for the unidimensional analysis are reported in Table 14.

As can be seen from Table 14, the chi-square obtained for the unidimensional model yielded a value of 1.14 with two degrees of freedom. The *p*-value for this test is .56, indicating an adequate fit of the data. The CFI value is 1.00, which is above the standard of .95; the values for RMSEA are not available. On the whole, the fit indices suggest very good model fit.

Table 13.

Standardized Coefficients of Rule Application Items under the Same Factor

Items	Coefficients
1	.83**
2	.87**
3	.83**
4	.78**
5	.79**
6	.65**

**$p < .001$

Table 14.

Goodness-of-Fit Indices for Null and Alternative Factor Models of Figure Detection Test

Factor Model	*df*	$\chi 2$	*p*-value	CFI	RMSEA	WRMR
Baseline model	6	28.91	.000			
Unidimensional model	2	1.14	.563	1.000	—	—

Standardized factor coefficients of each item were significant at .01 levels (see Table 15). However, the standardized item loadings on this factor were relatively low, indicating that the convergent validity of the test scores was not good.

Table 15.

Standardized Coefficients of Figure Detection Items under the Same Factor

Items	Coefficients
1	.36**
2	.43**
3	.55**
4	.57**

**$p < .001$

Construct Analysis for Working Memory Test

Two formats of Short Term Working Memory items were used in the test; namely, Binary Number Working Memory items and Shape Working Memory items. Two measurement models were tested for this test; namely, a unidimensional model with the two formats of items represented by a single factor and a first-order oblique model with two separate factors. Since a second-order CFA with two first-order factors and one second-order factor is always under-identified due to the absence of a constraint on the higher-order loadings, the second-order factor model was not tested here. Table 16 shows the fit statistics for the models.

Table 16.

Goodness-of-Fit Indices for Null and Alternative Factor Models of Working Memory Test

Factor Model	*df*	χ^2	*p*-value	CFI	RMSEA	WRMR
Baseline	24	359.813	.000			
Unidimensional model	22	51.868	.000	.911	.064	1.144
Two-factor oblique model	21	20.667	.480	1.000	—	.726

The results of the unidimensional model do not show a good fit. The chi-square value of the unidimensional model is 51.86 with 22 degrees of freedom. The low *p*-value $< .0001$ shows that the data did not fit the model well. The two-factor oblique model had an overall good fit with chi-square value of 20.667 ($df = 21$) and $p = .480$. This model was also found to have excellent goodness-of-fit indices: CFI = 1.000 and WRMR=.726.

Table 17 illustrates the standardized factor coefficients of the Working Memory test. Convergent validity is shown by the significant loadings of items on the underlying factors.

The intercorrelation between the factors of Binary and Shape is .59 with standard error of .04. The confidence interval for this correlation ranges from .51 to .67, which suggests acceptable discriminant validity for the Working Memory test scores.

Results for Each Research Question

Research Question 1: Which of the Alternative Models is the Most Valid Representation of the Domain of Matrix Reasoning Problem Solving?

Construct analysis results for each test were applied to prepare for the data used to conduct the SEM analysis. For tests with multiple factors (e.g., the Matrix Reasoning

Table 17.

Standardized Factor Coefficients on Working Memory Test for Each Identified Factor across Alternative Factor Models

Items	Unidimensional	Two-factor Oblique
Binary		
1	.45**	.53**
2	.38**	.44**
3	.46**	.57**
4	.57**	.66**
5	.56**	.68**
Shape		
1	.58**	.61**
2	.67**	.71**
3	.77**	.83**
4	.61**	.66**

$^{**}p < .001$

Test, the Rule Application Test, and the Working Memory Test), scales were created by adding up items scores that measure a common underlying factor. To increase the stability of the parameter estimates, a parceling method was applied for tests with only one dimension (e.g., the Rule Induction Test and Figure Detection Test). Holt (2004) recommended that binary items can be parceled by combining items with opposite item difficulties. Since the data in this research are dichotomous, Holt's suggestion was adopted. For example, In Figure Detection Test, item 1 (mean = .40) and item 4 (mean = .66) were combined and item 2 (mean = .47) and item 3 (mean = .49) were combined. There were two parcels for each unidimensional test.

Overall measurement model examination. The first step of the SEM analysis was to test the measurement model. This measurement model includes five latent variables as in Figure 8, with 11 variances for measurement errors, 11 factor loadings, and 10 factor correlations. Because there were at least two indicators for each factor, this model was treated as identified.

There is adequate evidence that the measurement model is a good fit. Therefore, the causal model may be tested through SEM modeling. Using the WLSMV estimation method, the measurement model was tested against the data.

As shown in Table 18, the chi-square test for the overall measurement model fit produced a value of 41.168 with 34 degree of freedom. The corresponding *p*-value of the chi-square test was .186. Goodness-of-fit indices for the measurement model yielded a CFI of .991, RMSEA of .025, and SRMR of .032. These indices meet the joint criteria of fitness evaluation, and indicated a reasonable model fit.

Mode comparisons. A key feature of SEM is the ability to explore causal relationships among latent variables. The hypothesized causal relationships of the latent variables are demonstrated in Figures 5 to 7. All of the three models were tested using the ML estimation method. Results of the model testing are shown in Table 18. None of the three hypothesized models had a reasonable fit with the actual data. Rule application as a mediator model yielded a chi-square value of 79.382 with 37 degrees of freedom,

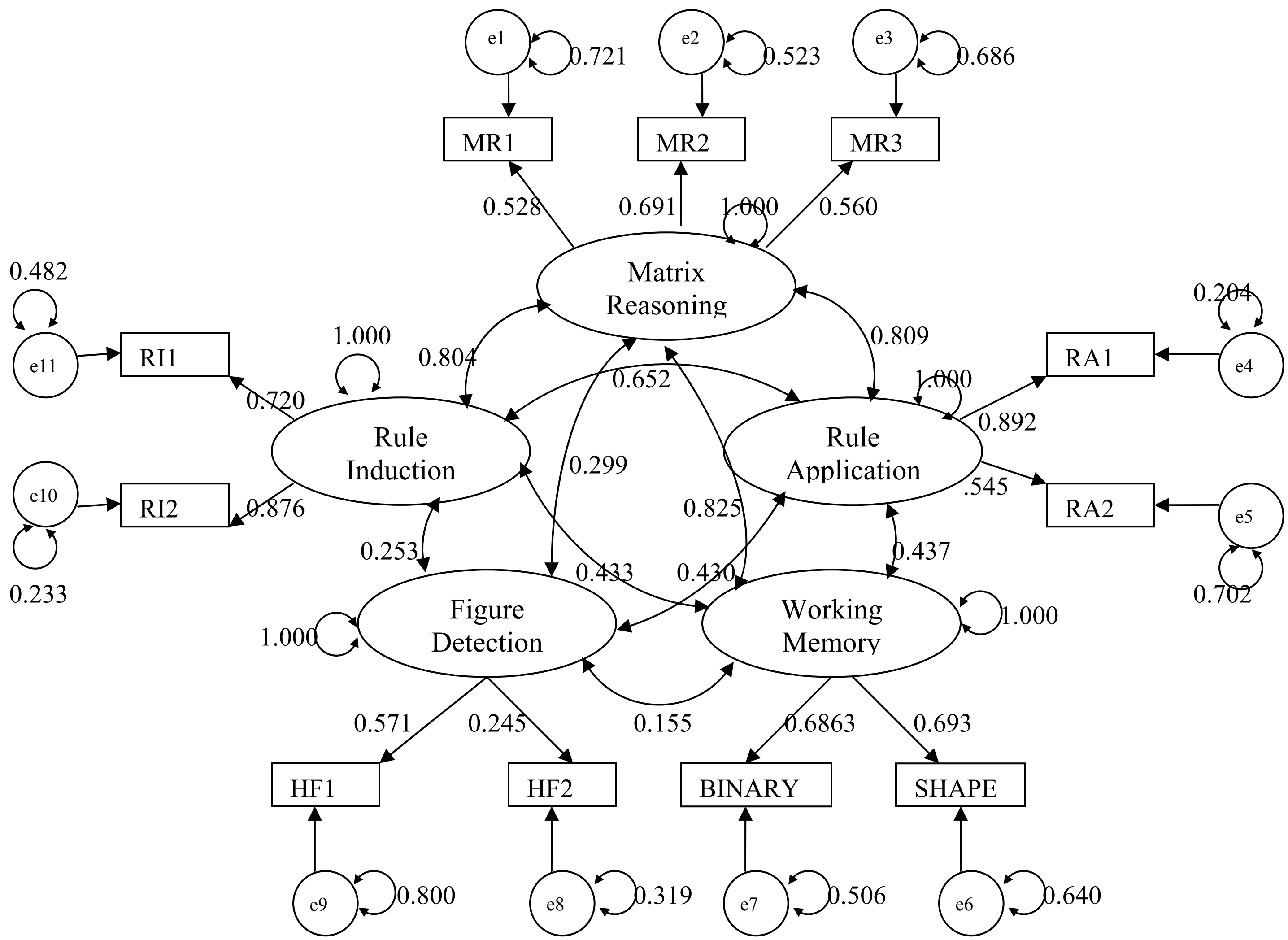


Figure 8. CFA measurement model of Matrix Reasoning problem solving.

producing a *p*-value less than .0001. Goodness-of-fit indices and CFI values for the three models were .950, the SRMR was .091, and the RMSEA was .59, offering further evidence that these models did not fit the data well. Although the goodness-of-fit indices for the other two models were acceptable, the chi-square *p*-values (< .05) showed poor model fit against the actual data. In conclusion, the three specified models were not valid representations of the domain constructs.

Compared with the three unreasonable hypothesized models, the measurement model generated better model fit, suggesting that more relationships among the latent variables should be specified in the model.

Table 18.

Hypothesized Model Comparison

Factor Model	*df*	χ^2	*p*-value	CFI	RMSEA	SRMR
Baseline	55	895.2	.000			
Components model	37	67.91	.001	.963	.05	.045
Measurement model	34	41.168	.186	.991	.025	.032
Process model	37	68.21	.001	.963	.05	.044
Rule application as mediator model	37	79.38	.000	.950	.06	.091

Research Question 2: What Modifications Can Be Made to Improve the Preferred Model?

As concluded from the results of question 1, all three models should be respecified. The MODINDICES function from MPLUS OUTPUT suggests what additional paths, means, intercepts, or variance components estimated in the model need to be freed in order to improve the model fit. The three models were respecified based on these suggestions.

Modification suggestions for the latent variables in the components model are listed in Table 19. The ON STATEMENT suggests that a path should be added from rule application to matrix reasoning. Since matrix reasoning is an endogenous variable, this suggestion does not satisfy the theory. Regardless, no further actions were taken to respecify the component model.

Table 19.

Modification Suggestions from MPLUS for Component Model

Statements	M.I.	E.P.C. S	Std E.P.C.	StdYX E.P.C
ON Statement				
RAPPLY ON MATRIX	11.13	-5.56	-1.22	-1.22

MODINDICE function suggestions for the process model are listed in Table 20. Among these suggestions, only the path from figure detection to rule application was theoretically acceptable. The respecified model was estimated with the same data. It yielded a chi-square value of 50.718 with a degree of freedom of 36. The corresponding *p*- value was .053, which was too high (>.05) to reject the null hypothesis that the model was a perfect model. Respectively, the goodness-of-fit indices are: CFI = .982, SRMR = .034 and RMSEA = .035, with a 90% confidence interval from 0 to .056. These index values are all favorable, indicating that the respecified model is acceptable.

Table 20.

Modification Suggestions from MPLUS for Process Model

Statements	M.I.	E.P.C. S	Std E.P.C.	Std YX E.P.C
ON Statements				
RID ON RAPPLY	17.22	-.50	-1.64	-1.64
RID ON MATRIX	27.16	-4.57	-3.32	-3.32
FD ON RAPPLY	17.22	.16	.61	.61
FD ON MATRIX	15.41	1.31	1.13	1.13
RAPPLY ON HID	17.22	1.23	.32	.32
WITH Statements				
RAPPLY WITH RID	17.23	-1.05	-.83	-.83
RAPPLY WITH HID	17.22	.33	.31	.31

For the rule application as mediator model, modification suggestions are shown in Table 21. Besides the paths of Rule induction on working memory and rule induction on figure detection, adding all other paths in the table would cause the model to have feedback loops. Since the original model was hypothesized as recursive, all paths which will lead to reciprocal direct effects would be omitted. Thus, only two paths, from working memory to rule induction and from figure detection to rule induction, were added back to the model. Estimation for the respecified model showed adequate model fit: Chi-square was 43.726 with 35 degrees of freedom, $p = .148$, CFI = .990, SRMR = .036 and RMSEA = .027, with a 90% confidence interval of 0 to .050.

Table 21.

Modification Suggestions from MPLUS for the Rule Application as Mediator Model

Statements	M.I.	E.P.C. S	Std E.P.C.	StdYX E.P.C
ON Statements				
MEMORY ON RID	24.17	.42	.41	.41
MEMORY ON RAPPLY	26.62	.21	.67	.67
MEMORY ON MATRIX	26.21	.86	.59	.59
RID ON MEMORY	24.17	.39	.41	.41
RID ON FD	11.13	.24	.24	.24
RID ON RAPPLY	29.72	.32	1.07	1.07
RID ON MATRIX	30.55	.98	.70	.70
FD ON RID	11.13	.24	.24	.24
FD ON RAPPLY	13.58	.13	.44	.44
FD ON MATRIX	13.68	.43	.31	.31
WITH Statements				
RID WITH MEMORY	24.17	.15	.40	.40
FD WITH RID	11.12	.08	.24	.24

Values of selected fit indices for the two acceptable respecified models both met the goodness-of-fit criteria. To choose the better model, the Akaike information criterion (AIC) was used. AIC was developed by Hirotsugu Akaike (1974). It can be used to compare nonhierarchical models from the same data set. AIC indicates model fit and model parsimony. One of the ways to calculate AIC is AIC = $\chi^2 + 2q$, where q is the number of free model parameters. Respectively, the *q* value for process model and rule application as mediator model is 30 and 31. The model with the smaller AIC will generally have a better model fit and will be most likely to replicate. For the respecified process model, AIC = 50.718 + 2 * 30 = 110.718; and for the respecified rule application as mediator variable model, AIC = 43.726 + 2 * 31 = 105.73. The latter model was kept due to its lower AIC value. This decision was confirmed by comparing other Goodness-of-fit indices of the two models (see Table 22). The final model and its coefficients were demonstrated in Figure 9.

It is notable that a satisfactory model was found that fits the data according to the standards set in advance. The model fits with no pathway from working memory to figure detection, which is consistent with other literature that some matrix items require a separate visuospatial factor. The model shows a modest pathway from figure detection to rule application, which makes theoretical sense. The pathway from figure detection to matrix reasoning, however, is close to zero, showing that the hypothesized construct relevance of figural decomposition to matrix reasoning was not demonstrated by this otherwise well-fitting model.

Table 22.

Goodness-of-Fit Indices for the Respecified Models

Factor Model	*df*	$\chi 2$	*p*-value	CFI	RMSEA	SRMR
Respecified process model	36	50.718	0.053	0.982	0.035	0.034
Respecified rule application as mediator model	35	43.726	0.148	0.99	0.027	0.036

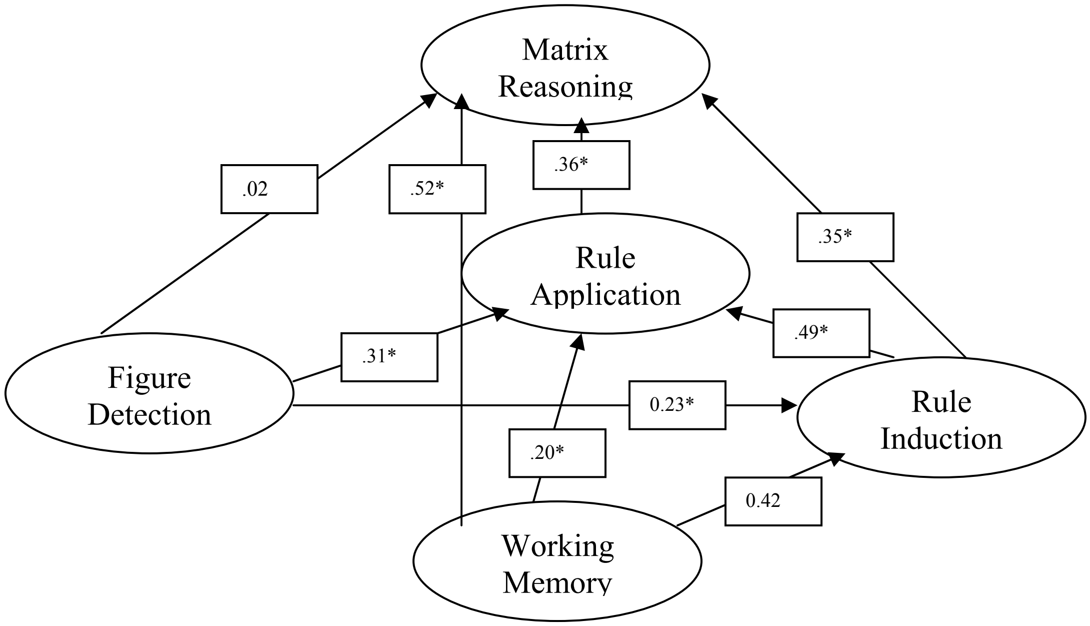


Figure 9. The standardized estimation of the respecified model that empirically fits the actual data.

Analysis of the data and reflection on what it means led to some grave concerns regarding the Figure Detection Test, and thus about the latent variable it is supposed to measure. These concerns deal with score reliability and convergent validity and should be discussed in more detail.

The first concern focused on reliability and validity issues. As reported at the beginning of the results analysis, the Cronbach's alpha coefficient for the Figure Detection Test score was .53, which is lower than the acceptable value of .70. The lower coefficient indicates the inconsistency among the individual items in this scale. Although the CFA analysis showed a valid unidimensional construct, the relatively low alpha demonstrated a high degree of random error in item scores. Evidence of convergent validity was also unsatisfactory for this test because item loadings on the same factor were relatively low, showing that the items were not theoretically intercorrelated in an acceptable manner. The low reliability and unsatisfactory convergent validity results of the Figure Detection Test make its further application in the study unwise.

The second concern about Figure Detection Test relates to its low correlation with other latent variables. The correlation matrix is reported in Table 23. As shown in the table, when compared with other correlation coefficients, figure detection has the lowest correlation of any variable. This suggests the construct irrelevance between figure detection and other latent variables in the same model.

The third reason for concern about the Figure Detection Test is construct relevance to the constructs in play within this entire test battery. The Figure Detection Test was selected for this research to test students' figure decomposition ability. A

Table 23.

Correlation Matrix for the Latent Variables in the Model

	Working Memory	Rule Induction	Figure Detection	Rule Application	Matrix Reasoning
Working Memory	1.00				
Rule Induction	.43	1.00			
Figure Detection	.16	.25	1.00		
Rule Application	.44	.65	.43	1.00	
Matrix Reasoning	.83	.80	0.30	.81	1.00

well known existing test, Hidden Figure Test, was selected. In this test, the students were asked to find a picture in a very complicated background that matched one of the listed pictures in the answer options. A further investigation on each item in this test showed that student mean score for every item was relatively low, indicating that the items were difficult for the sample of students who participated in the research. An additional source of grave concern about the construct match between hidden figures and matrix reasoning is that not all of the Matrix Reasoning items require students to possess even a moderate level of figure decomposition ability. More than half of the items had low visual complexity. Therefore, the overly difficult Figure Detection items were not strongly relevant to overly easy Matrix Reasoning problems.

Based on the analysis above, a model without the figure detection variable in the analysis should be examined. The following analyses were redone by dropping the figure detection variable from the model.

In the respecified measurement model without the variable of figure detection, there were only four Latent variables: (a) matrix reasoning, (b) rule application, (c) rule induction, and (d) working memory. The same estimation method was used as the measurement model in research question 1. The model testing results are shown in Table 24.

Table 24.

Goodness-of-Fit for Measurement Model with Figure Detection Omitted

Factor Model	*df*	$\chi 2$	*p*-value	CFI	RMSEA	SRMR
Baseline	36	799.51	.000			
Measurement model	21	27.65	.150	.992	.031	.031
Structural equation model	21	27.65	.150	.992	.031	.031

Measurement model and SEM for this analysis yield the same results. The results show satisfactory model fit. The chi-square test for the overall measurement model fit produced a value of 27.65 with 21 degrees of freedom. The corresponding *p*-value for the chi-square test was .150. Goodness-of-fit indices yielded a CFI of .992, RMSEA of .031, and SRMR of .031. Even though the model fit with figure detection was not bad, this one fits even better. Estimations are given in Figure 10.

Research Question 3: What Are the Significant Direct and Indirect Effects of the Latent Variables?

Direct and indirect path effects from Figure 10 are reported as follows.

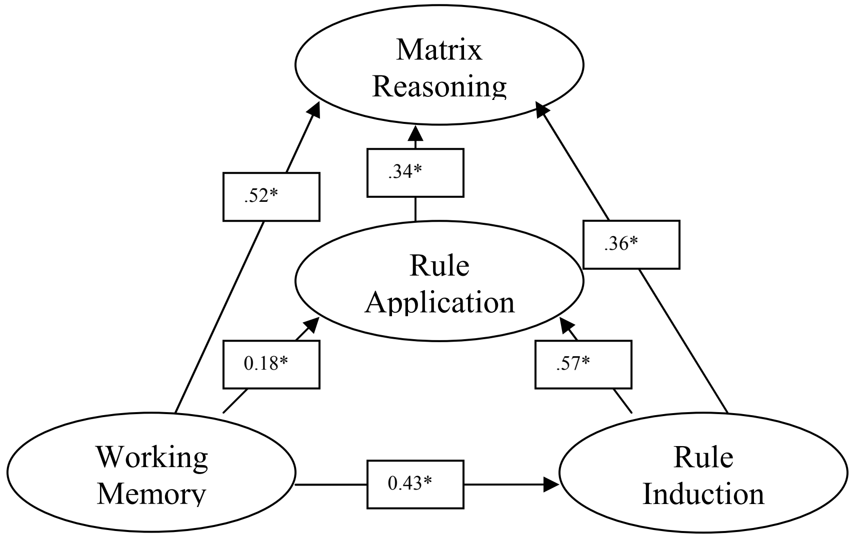


Figure 10. The standardized estimation for the respecified model with the figure detection variable omitted.

Direct effects. The direct and indirect effects of the acceptable model respecified in question 2 are delineated in Table 25. The unstandardized path coefficients, standard error, ratio of estimation to standard error, and the standardized coefficients are reported column by column. The unstandardized estimate represents the amount of change in the outcome variable as a function of a single unit change in the variable causing it. The estimate divided by the standard error, which indicates a significant effect when it is larger than 1.96, tests the significance of the parameter estimation. As shown table 25, besides the paths from figure detection ability to matrix reasoning and from working memory ability to rule application ability, all other paths in the model have significant direct effects (ratio values are larger than 1.96).

To examine the relative strengths of association across these latent variables that are measured using different scales, the standardized coefficients were compared. Standardized coefficients represent the amount of change in an outcome variable per standard deviation unit of a predictor variable. For example, the standardized path coefficient from working memory to matrix reasoning is .52. This means that if working memory ability increases by one standard deviation from its mean, matrix reasoning ability would be expected to increase by .52 of its own standard deviations from its own mean while holding all other relevant connections in the region constant.

Clearly, the relationship of the three latent variables to matrix reasoning in descending order of magnitude of the standardized coefficients are working memory ability (.52), rule induction ability (.34), and rule application ability (.36). This finding demonstrates that the working memory ability is more effective at explaining the shared variance of matrix reasoning than other latent variables.

Table 25.

Results of Standardized Direct and Indirect Effects of the Respecified Model

Effects	Estimates	S.E.	Est./S.E.	Std
Direct Effects of the Latent Variables				
working memory → matrix reasoning	.38	.09	4.12	.52
rule induction → matrix reasoning	.27	.08	3.50	.36
rule application → matrix reasoning	.08	.02	3.14	.34
working memory → rule application	.62	.30	2.07	.18
rule induction → rule application	1.96	.27	7.38	.57
working memory → rule induction	.42	.10	4.08	.43
Indirect Effects from working memory to matrix reasoning				
working memory → rule induction → matrix reasoning	.11	.04	3.14	.15
working memory → rule application → matrix reasoning	.05	.02	2.02	.06
working memory → rule induction → rule application → matrix reasoning	.06	.03	2.46	.08
Indirect Effects from rule induction to matrix reasoning				
rule induction → rule application → matrix reasoning	.15	.05	2.82	.20

Significant effects also exist in the paths from rule induction ability to rule application ability, from working memory ability to rule application ability, and from working memory ability to rule induction ability.

Indirect effects. Indirect effects were investigated from the two exogenous variables working memory and rule induction to the predicted variable of matrix reasoning. Results from Table 25 show significant indirect effects of working memory through mediating variables. These three significant paths listed below.

1. working memory → rule induction → matrix reasoning
2. working memory → rule induction → rule application → matrix reasoning
3. working memory → rule induction → rule application → matrix reasoning

The significant indirect effects partially indicate the flow path of Matrix Reasoning problem solving. Also, there is a significant indirect path from rule induction to matrix reasoning through rule application.

Chapter 5: Conclusion and Discussion

The purpose of this research was to gain a deeper insight into the domain of matrix reasoning. An attempt was made to develop a valid model to understand the construct of matrix reasoning. In this SEM model, subabilities involved in solving Matrix Reasoning tasks were postulated and the relationships among the subabilities were sufficiently hypothesized. The results explain how typical students in this sample solve Matrix Reasoning problems. This model provides an explanation supported by empirical data. The results have implications for designing Matrix Reasoning Tests and instruction.

Before discussing the results, the issue of how the results of SEM are to be understood must be addressed. As mentioned in the methodology section of this work, the SEM technique is used to examine hypothesized causal relationships among variables with a linear equation system and to test whether or not the actual data is consistent with the model. However, the SEM technique can not prove the causal model. When the hypothesized model is accepted, the model fit data only support the argument that the model is a valid representation of the relationships among variables. However, there may be several alternative models that could fit the data equally well. When the hypothesized model is rejected, the model is demonstrated as definitely not fitting the data.

Issues Regarding the Figure Detection Test

The Figure Detection Test studied in this research was designed to measure a hypothesized construct, figural decomposition ability, which refers to find corresponding figures embedded in complex figures. After reviewing the literature, I found that the hidden figure task can function as the instrument to measure figure decomposition ability because hidden figure task involve figure-ground segregation. This task requires observers to identify the given simple figures hidden in a complex visual configuration. Hidden figure tasks also require visual decomposition ability.

However, according to the results of the analysis, figure detection had no significant direct effect on matrix reasoning, nor did it show any significant indirect effects through mediator variables. This seems to demonstrate that the figure decomposition ability does not play a crucial role in matrix reasoning. This result rejected the hypothesis that figure decomposition ability has a significant effect on matrix reasoning. This result is consisted with Tsakanikos and Reed's research (2003). In their study, Tsakanikos and Reed found that although there was a correlation between the number of correct responses on Raven's Progressive Matrices and Hidden Figures Test, they maintained a substantial amount (> 95%) of non-shared variance. Even so, any interpretation of this result should consider the following issues.

First, this finding is suspect because the score reliability and validity of the Figure Detection Test were less than satisfactory. As previously discussed, Cronbach's alpha coefficient for the Figure Detection Test was .53, revealing a less than acceptable level of consistency of the items in this test. In addition, the standardized factor loadings for each item were too low to demonstrate satisfactory discriminant validity. Also, more than half of the 14 Matrix Reasoning items did not require figural decomposition ability. The two reasons listed above are both due to the insufficiently construct-sensitive test design. Consequently, all results related to the figure detection variable should be interpreted with caution.

Second, the results prompted the researcher to reflect on whether or not the Hidden Figure task was a good format to measure figure decomposition ability. While the Hidden Figure task requires subjects to visually encode and parse the figures so that one can find the correct figure in a complicated figural background, it cannot test the process of finding correspondence elements which requires subjects to ascertain which figure elements in the three entries in a row are related. Therefore, a conservative conclusion may be drawn from the results; namely, that whatever construct was measured by the Figure Detection Test has no direct or indirect effects on whatever constructs are measured by the Matrix Reasoning Test. This result does not constitute proof that visual encoding and parsing, appropriately measured, are not important parts of Matrix Reasoning problems. Also, we must keep in mind that this conclusion is reached based on the finding that the reliability and validity of Figure Detection Test scores. There is also evidence that even with its unsatisfactory psychometric properties, figure detection did relate to rule application ability.

Which tasks in what test format would be effective for assessing the figural detection construct and its relationship to correspondence finding? This is a question for future research, informed perhaps by the off-target construct-relevance of part of this study. Theoretically, correctly testing figure decomposition ability can help future researchers to answer the remaining questions in this research.

Finding or developing better measures of figural decomposition ability may have practical benefits worth pursuing. Some learners who can not successfully finish the Matrix Reasoning tasks may need deeper diagnosis involving figural decomposition ability and accurate correspondence finding.

Discussion

As discussed above, figure detection is not a satisfactory variable in this research. More than half the Matrix Reasoning problems did not require the figure decomposition ability. Therefore, it was reasonable and appropriate to drop the figure detection variable from the analysis of the final model.

The three questions addressed in Chapter 4 can be condensed into two to facilitate discussion. First, is there a valid model representation to demonstrate the relationships between identified abilities that are applied to solve Matrix Reasoning problems? Secondly, if there is a valid model representation, what are the significant direct and indirect relationships between the variables in this model?

In regards to the first question, SEM analysis shows that there is a valid model which can demonstrate the relationships between latent variables. However, this valid model is not one of the three hypothesized models. Rather, it is a combination of the three models. In the first hypothesized model, working memory was the basic variable. In addition to its direct effect on matrix reasoning, it also had indirect effects through rule application and rule induction. However, there was no relationship specified between rule induction and rule application. The second model was the rule application as the mediator variable model. In this model, all of the variables had a direct effect on matrix reasoning. Also, rule induction and working memory had indirect effects on matrix reasoning through rule application. The third model specified the path of working memory→rule induction→rule application→matrix reasoning. It also set paths from working memory to rule induction and rule application. Results show that none of the three models were a sufficient fit to the data. Paths should be set among all the predicted variables.

The second question will be answered by examining the relationships among the variables.

The Relationship between Matrix Reasoning and the Other Three Subabilities

In this study, paths from working memory, rule induction, and rule application to matrix reasoning ability were defined in the model. All three paths showed significant effects. The relative strength of the relationships can be evaluated by examining the standardized coefficients. The strongest relationship existed between working memory and matrix reasoning ability, with a standardized coefficient of .52. The relationships between matrix reasoning ability and rule induction ability and between matrix reasoning ability and rule application ability dropped to .36 and .34 respectively. These results provide evidence that working memory is the most important factor which affects subjects' performance in solving Matrix Reasoning problems. This result is consistent with Lohman's (2001) study of the relationship between working memory and reasoning in which he discussed that "although many different processes may be executed in the solution of a task, individual differences in them may primarily reflect individual differences in working memory resources" (p. 223). The relationship between working memory and matrix reasoning will be further examined.

The Path from Working Memory to Matrix Reasoning

As mentioned above, a significant and substantial causal path was found from working memory to matrix reasoning. The matrix reasoning ability increased .52 of its own standard deviations from its own mean. Working memory ability increased by one standard deviation from its mean while holding all other relevant regional connections constant. Moreover, indirect effects were also established from working memory to matrix reasoning. Two variables emerged as significant mediators between working Memory and matrix reasoning. Subjects who have more working memory ability are likely to perform better on Rule Induction Test; this in turn increases the likelihood they will perform better on the Rule Application Test, which leads to an increase in their scores on the Matrix Reasoning Test. This is an important process. The ability at the lower order of the path will not only affect performance on solving Matrix reasoning problems, it will also affect other subablities in the higher order of the path. Although the majority of the effect is explained by the direct effect, the indirect effects lead to the interpretation that working memory ability is a component that improves performance on both rule induction and rule application, and thus on matrix reasoning.

Study Contributions

This study yields a deeper insight into the domain of matrix reasoning. Previous research has focused more on the item (task) side than on the thinking person side in attempts to understand the domain of matrix reasoning. Starting from the process of how a thinking person solves a matrix reasoning problem, this research identified four subablities which were hypothesized to significantly contribute to matrix problem solving. By applying the technique of structural equation modeling, variable relationships which were hypothesized in the models were tested and respecified. The final respecified model with Figure Detection Test omitted was demonstrated to be a valid representation of the domain; this model was able to fit well with actual data. This research provided evidence that working memory ability is the most important resource that can be used to explain individual differences in Matrix Reasoning problem solving. The rule induction ability and rule application ability are also significantly and directly connected to matrix reasoning. Furthermore, significant indirect effects from working memory to matrix reasoning through variables of rule induction and rule application indicted that there was a valid problem solving process path that existed, although the indirect effect was much smaller than the direct effects. These results provide a different and personal view to the understanding of the domain of matrix reasoning. By and large, matrix reasoning cannot be explained by any process independently; it is a combination of working memory, rule induction, and rule application, and perhaps of other processes not demonstrated in this study.

This research did not definitively rule out figural decomposition as an important part of the process. When matrix problems are used with a strong requirement for figural decomposition ability, it is still possible that more figurally demanding matrix problems and a valid and reliable figural decomposition test would show an important relationship.

Implications for Training

Inductive reasoning is considered to be the central part of fluid intelligence. Kauer (2002) stated, "Inductive reasoning enables one to detect regularities, rules, or generalizations and, conversely, to detect irregularities. This is one way in which we structure our world" (p. 1). Obviously, improving inductive reasoning ability will allow for better learning and living in the real world. Studies have shown that fluid ability can be enhanced with training (e.g., Deeny & Heidrich, 1990; Welko & Johannes, 1997). A large-scale study by Csapo (1997) has confirmed that inductive reasoning correlates substantially with school achievement. Training significantly improves performance. For students, training on inductive reasoning will not only foster their competencies in intelligence, but also will help them improve their academic learning.

Previous training methods for Matrix Reasoning problems included demonstrating the problem solving process to subjects and then asking them to repeat the process by using the same strategy (Deeny & Heidrich, 1990). This study provides a potential method for individualized training. Students may be evaluated by the three tests developed in this research to diagnose the problems they may encounter as they try to improve their fluid intelligence. (If the Figure Decomposition ability is counted, then there are four tests; however, this fourth test requires more development and research.) Training should ideally be provided based on individual need. After training on subablities, a problem solving process then will be demonstrated. The learning process is a circle; diagnosis, training, and evaluation should be combined.

Limitations of the Research

Perhaps the most serious limitation of this research is that the measurement error from the Figure Detection Test put a brake on our theoretical interpretation and empirical inference. This is especially true for those Matrix Reasoning questions which have complex figure combinations. It is confirmed that figure complexity was an important resource to explain item difficulty differences. Figure decomposition ability therefore should play an important role in solving Matrix Reasoning problems. Unfortunately, our Figure Detection Test was not able to provide strong evidence to either reject or accept this hypothesis.

An issue worthy of consideration is that the Figure Detection Test does not solely test the Figure Decomposition ability. We were not able to find an appropriate task format to test the ability of both parsing complex figures and finding corresponding elements. Also, as shown from the research, typical hidden figure tasks proved to be an unsatisfactory task format to test the ability of parsing figures. These limitations prevented the further understanding of how the figure decomposition construct should affect matrix reasoning problem solving.

Another limitation of this study is related to external validity. Although the two schools that participated in this research were randomly selected, they cannot represent the entire population of either city or rural area. The homogeneity of the sample also brought further potential limitations to generalizability. Our samples were groups of students of ages 13 to 15 solely of Chinese nationality. Similar findings and results may not be guaranteed from a sample more diverse with regard to age or ethnic identity.

Future Research

A continued discussion of the limitations of this research, future research, and data collection efforts will require refinements in current theories; a number of practical and theoretical questions will be addressed.

Theoretically, future research should further explore whether a valid test is available to measure figure decomposition ability. Results from the present research showed that the Hidden Figures Test can not measure figure decomposition appropriately. In addition to figural recognition, figure decomposition ability also requires the ability to find correspondence elements from these figures. A valid Rule Decomposition Ability Test should measure these two components. Furthermore, if such a valid test exists, how would the new model including this test differ from the one estimated in this research?

The sample in this research consisted of students from China between the ages of 12 to 15 inclusively. How would the model change to if responses from a larger and more diverse sample were investigated? Would a different model fit better for other age levels or cultural groups? These questions need to be investigated in future studies.

Future research on practical considerations could include the following.

1. How well do the tests developed in this research function when used for diagnostic and training purposes?

2. Would training in matrix reasoning that focused on using the subability tests developed in this study be more or less effective than using the training method consisting of demonstrating the problem solving procedure, as done in previous research?

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Appendix A

Informed Consent Form for Focus Group Participants

Participants
Informed Consent Form
(For principals and teachers)

Introduction
This research is being conducted by Kairong Wang under the help of Dr. Richard Sudweeks. Kairong Wang is a graduate doctoral student in Department of Instructional Psychology and Technology of Brigham Young University. The purpose of this research is to investigate how people solve the Matrix Inductive Reasoning problems. Your students are invited to participate in this study.

Procedure
The students in your school from grade 6 to grade 8 will invited to complete a test online. The test is used to assess general intelligence. It includes five parts. Individually, they are Working Memory test, Rule Induction test, Rule Application test, Hidden Figure test, and Matrix Inductive Reasoning test. The test consists of 46 items and will take approximately 45 minutes.

Risk/Benefits
There are minimal risks for participation in this study. However, the students may feel frustrated when they are not able to complete some of the items.

The benefits to the subjects will be that they have a chance to have a brain exercise. It is hoped that through the students response researcher will be able to learn more about the structure of matrix reasoning ability. It will also bring ideas about how to help the students to improve their reasoning ability.

Confidentiality
The test is online. Students' response will be saved in the database directly. Other than those who directly involved with the research, nobody else will be able to access the data. The students' identities such as name, date of birth, school achievement are not required. The results will only be reported as group data with no identifying information.

Compensation
Compensation to the students will be the report of results from this research. Further help will be provided under the class teachers' request.

Participation
Participation in this research is voluntary. You and the students have the right to withdraw at anytime or refuse to participate for any reason without penalty and without affecting their school standing..

Questions about the Research
If you have any questions regarding this study, please contact Kairong Wang at 1-858-566-6475, kairong_wang2003@yahoo.com or Professor Richard Sudweeks, (801) 422-7078, Richard_Sudweeks@byu.edu .

Questions about your Rights as Research Participants

1. If you have any questions regarding the rights as a participant in this research project, please contact the ORCA office in Brigham Young University. Christopher Dromey, PhD, IRB Chair; 422-6461; 133 LRB; christopher_dromey@byu.edu .

I have read, understood, and received a copy of the abvove consent and desire of my own free will to participate in this study.

Signature: ___________________________ Date: ___________________________
School: ___________________________

Participants
Informed Consent Form
(For students or parents)

Introduction
This research is being conducted by Kairong Wang under the help of Dr. Richard Sudweeks. Kairong Wang is a graduate doctoral student in Department of Instructional Psychology and Technology of Brigham Young University. The purpose of this research is to investigate how people solve the Matrix Inductive Reasoning problems. Your students are invited to participate in this study.

Procedure
You will be invited to complete a test online. The test is used to assess general intelligence. It includes five parts. Individually, they are Working Memory test, Rule Induction test, Rule Application test, Hidden Fgure test, and Matrix Inductive Reasoning test. The test consists of 46 items and will take approximately 45 minutes.

Risk/Benefits
There are minimal risks for participation in this study. However, the students may feel frustrated when they are not able to complete some of the items.

The benefits to the subjects will be that they have a chance to have a brain exercise. It is hoped that through the students response researcher will be able to learn more about the structure of matrix reasoning ability. It will also bring ideas about how to help the students to improve their reasoning ability.

Confidentiality
The test is online. Your response will be saved in the database directly. Other than those who directly involved with the research, nobody else will be able to access the data. Your identities such as name, date of birth, school achievement are not required. The results will only be reported as group data with no identifying information.

Compensation
Compensation will be the report of results from this research. Further help will be provided your request.

Participation
Participation in this research is voluntary. You have the right to withdraw at anytime or refuse to participate for any reason without penalty and without affecting their school standing..

Questions about the Research
If you have any questions regarding this study, please contact Kairong Wang at 1-858-566-6475, kairong_wang2003@yahoo.com or Professor Richard Sudweeks, (801) 422-7078, Richard_Sudweeks@byu.edu .

Questions about your Rights as Research Participants
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I have read, understood, and received a copy of the abvove consent and desire of my own free will to participate in this study.

Signature: ___________________________ Date: ___________________________

Letter of Statement from Principal

This is a letter to state that as a principal of Wangjing Middle School in Beijing, I have the legal guardianship of the students on participating in educational research while they are in the school.

I have read, understood, signed and received a copy of the consent form of the study from the researcher. The research of “Exploring the Domain of Geometric Inductive Reasoning Problems: A Structural Equation Modeling Analysis” by Kairong Wang is approved to be administrated in our school.

兹证明作为望京中学的校长, 我对我校学生参加在校教育科研有合法的监护权.
我校已经审查并通过王凯荣在我校进行“Exploring the Domain of Geometric Inductive Reasoning Problems: A Structural Equation Modeling Analysis”的研究申请.

签名: ______________________ 时间 ______________________
学校 ______________________

Appendix B

Statistics for Each Item in the Tests

Tests	Items	Mean	Sd	Item-total Correlation Coffiecient	Alpha if Item Deleted
Matrix Reasoning Test	1	.84	.36	.45	0.84
	2	.92	.28	.62	0.83
	3	.84	.37	.47	0.84
	4	.90	.31	.63	0.83
	5	.41	.49	.56	0.83
	6	.82	.39	.42	0.84
	7	.72	.45	.47	.84
	8	.70	.46	.62	.83
	9	.74	.44	.62	.83
	10	.74	.44	.46	.84
	11	.42	.49	.35	.85
	12	.24	.42	.23	.85
	13	.23	.42	.66	.83
	14	.49	.50	.44	.84
Rule Application Test	1	.60	.49	.36	.88
	2	.72	.45	.70	.86
	3	.43	.50	.48	.87
	4	.42	.49	.45	.87
	5	.67	.47	.69	.86
	6	.44	.50	.41	.88
	7	.69	.47	.74	.86
	8	.69	.47	.69	.86
	9	.57	.50	.85	.85
	10	.33	.47	.83	.85
	11	.33	.47	.24	.88
	12	.23	.42	.42	.87
Rule Induction Test	1	.78	.41	.75	.86
	2	.89	.31	.78	.86
	3	.85	.35	.75	.86
	4	.72	.45	.70	.87
	5	.77	.42	.73	.86
	6	.86	.35	.55	.89
Figure Detection Test	1	.40	.49	.30	.25
	2	.47	.50	.44	.29
	3	.49	.50	.49	.37
	4	.66	.47	.40	.37

(Table continues)

Table 7 (continued)

Tests	Items	Mean	Sd	Item-total Correlation Coffiecient	Alpha if Item Deleted
Binary Working Memory Test	1	.84	.37	43	.67
	2	.75	.43	35	.70
	3	.73	.44	51	.63
	4	.82	.38	49	.65
	5	.67	.47	51	.62
Shape Working Memory Test	1	.66	.47	.34	.63
	2	.40	.49	.40	.53
	3	.30	.46	.49	.49
	4	.27	.44	.40	.55